**Chapter 2 – Formulas and Decimals**

**Section 2A – Rounding, Comparing, Adding and Subtracting Decimals**

Look at the following formulas. The first formula (P = A + B + C) is one we use to calculate perimeter of a triangular region where A, B and C are the lengths of the sides of the triangle. Suppose we want to find the perimeter of a triangle with sides 3.1 cm, 2.67 cm and 4.129 cm. Since many of the numbers we want to plug into formulas involve decimals, we need to be able to add and subtract decimals.

As with whole numbers, the key to adding and subtracting decimals is place value. So let us review the place values of decimals. For example, look at the number 3.12589

3 **.** 1 2 5 8 9

Ones Tenths Hundredths Thousandths Ten-Thousandths Hundred-Thousandths

For example tenths means out of 10. So 0.1 means 1 tenth or 1/10. 0.02 means 2 hundredths or 2/100, and so on.

We cannot add or subtract the tenths place with the thousandths place. The place values have to be the same.

Another key to decimals is that you can add zeros to the end of a decimal and it does not change its value. For example 2.7 is the same as 2.70 which is the same as 2.700 which is the same as 2.7000 and so on. This can be helpful when lining up decimal places.

Comparing Decimals

Decimals are often difficult to compare. For example, in statistics we often have to compare a P-value decimal of 0.0035 to 0.05 and see which is larger. The key is to add zeros until both numbers end in the same place value.

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In the above example, to compare 0.0035 to 0.05 we can add two zeros to the end of 0.05. Remember this does not change its value.

0.0500

0.0035

Notice it is now easy to compare them. Which is larger, 500 or 35? Obviously 500 is larger than 35 so 0.05 is larger than 0.0035. (In statistics this tells us that the P-value is significantly low.)

Try another one. Compare 24.3 to 24.2997. Again if we add zeros to the 24.3 so that it ends in the same place value, we get the following.

24.3000

24.2997

Which is larger, 243000 or 242997? Since 243000 is larger than 242997, we see that 24.3 is larger than 24.2997.

Try the following examples with your instructor.

Example 1: Which is larger, 0.004 or 0.03? Example 2: Which is larger, 1.0249 or 1.02?

Rounding Decimals

To correctly round a decimal, we need to decide to which place value we want to round to. We then look to the right of that place value. If the number to the right is 5 or above, we will round up by adding one to the place value. If the number to the right is 4 or below, we will round down by leaving the place value alone. The key to remember is that after rounding, we want to stop after the place value so that the number is simpler to use.

For example, round 0.03792 to the hundredths place. The hundredths place is the 3. Looking to the right of it we see a 7. Since 7 is large (5 or above) we round up by adding 1 to the place value. So the 3 becomes 4 and we cut off the rest of the numbers.



Let’s try another example. Round 1.83597 to the tenths place. The tenths place is the 8. Looking to the right of it we see a 3. Since 3 is small (4 or below) we round down by leaving the place value alone. So the 8 stays as it is and we cut off the rest of the numbers.



Try the following examples with your instructor. Pay close attention to the place values.

Example 3: Round 3.986114 to the Example 4: Round 0.74386 to the
 thousandths place. ones place.

Example 5: Round 72.973 to the
 tenths place.

Addition and Subtraction

Let’s look at an example. Let’s add 3.258 + 7.9 ; It is important to line up the place values before adding or subtracting. Let us rewrite 7.9 as 7.900 so that both numbers end in the thousandths place. Now we add exactly as if the numbers were whole numbers.



Similarly to subtract, we can add zeros to make sure the numbers end in the same place value. Now we can line up the place values and subtract. Let’s subtract 9 – 3.2751 First let’s rewrite the 9 as 9.0000 so that the place values line up. Now subtract. Remember an easy way to borrow is to borrow 1 from 9000, which leaves 8999. Then add 10 to the last place value.



Try the next few examples with your instructor.

Example 6: 34.529 + 6.38 Example 7: 14.8 + 9.4726

Example 8: 2.4 – 0.853 Example 9: 134.6 – 89.7751

**Practice Problems Section 2A**

1. Which is larger, 0.082 or 0.0399? 2. Which is smaller, 1.13 or 1.028?

3. Which is smaller, 0.034 or 0.0099? 4. Which is larger, 34.5 or 34.499?

5. Which is larger, 0.05 or 0.00016? 6. Which is smaller, 8.719 or 8.7?

7. Which is smaller, 0.378 or 0.4? 8. Which is larger, 2.41 or 2.4086?

9. Which is larger, 0.0058 or 0.005? 10. Which is smaller, 12.6 or 12.499?

11. Which is smaller, 0.0175 or 0.00175? 12. Which is larger, 68.327 or 68.7?

13. Round 0.7829 to the hundredths place. 14. Round 3.529 to the ones place.

15. Round 9.647 to the tenths place. 16. Round 0.7956 to the thousandths place.

17. Round 0.58699 to the thousandths place. 18. Round 78.462 to the ones place.

19. Round 1.375 to the tenths place. 20. Round 0.7956 to the hundredths place.

21. Round 72.973 to the ones place. 22. Round 0.053289 to the thousandths

23. Round 0.053289 to the tenths place. 24. Round 8.00374 to the hundredths

25. Round 3.745117 to the ten-thousandths 26. Round $48.1277 to the nearest cent
 place. (hundredths place).

27. Round $359.0244 to the nearest cent. 28. Round $1372.631 to the nearest cent
 (hundredths place). (hundredths place).

29. Round $74.538 to the nearest dollar. 30. Round $671.284 to the nearest dollar
 (ones place). (ones place).

31. Round $481.723 to the nearest dollar. 32. Round $10,457.43 to the nearest dollar
 ones place). (ones place).

For #33-47, add the following decimals. Don’t forget to line up the place values.

33. 0.2 + 0.135 34. 0.89 + 1.35 35. 2.83 + 1.17

36. 12 + 17.431 37. 0.064 + 1.23 38. 13.9145 + 7.528

39. 3.73 + 13.898 40. 0.00827 + 1.443 41. 6.834 + 7.0029

42. 46.1+ 0.439 43. 3.408 + 13.93 44. 0.845 + 6.0851

45. 51.92 + 64.083 46. 0.0072 + 0.02665 47. 9.31 + 7.0917

For #48-62, subtract the following decimals. Don’t forget to line up the place values.

48. 8.4 – 6.9 49. 1.003 – 0.475 50. 9.31 – 6.4227

51. 14.2 – 4.75 52. 2.03 – 0.187 53. 7 – 2.581

54. 17 – 3.26 55. 4.931 – 2.1 56. 8.4 – 0.0035

57. 3.6 – 0.047 58. 0.835 – 0.3724 59. 0.9 – 0.638

60. 19.2 – 3.055 61. 18.528 – 17.9 62. 1.108 – 0.0266

For #56-61, add or subtract the following decimals in order from left to right.
Don’t forget to line up the place values.

63. 13.4 + 1.58 + 0.472 64. 8 + 2.14 + 6.9 65. 8.235 + 7.49 + 14

66. 4.75 – 1.327 + 0.8 67. 7.3 + 0.94 – 0.037 68. 127 – 33.9 + 4.82

A formula we have seen before is the profit formula P = R – C where P is profit, R is revenue and C is cost.

69. Find the profit if R = $178.14 and C = $95.77

70. Find the profit if R = $13000 and C = $2879.41

71. Find the profit if R = $5,264 and C = $3095.77

72. Find the profit if R = $7,851.26 and C = $5248.99

Another formula we have seen is the perimeter of a rectangular region in geometry.
P = L +W + L + W . (Use this formula for #73-76)

73. Find the perimeter of a rectangular field that is 1.4 miles by 0.85 miles.

74. Find the distance around the outside of a rectangular room that is 13.5 feet by 10.8 feet.

75. Find the perimeter of a rectangular window that is 17.5 inches by 13.85 inches.

76. Find the distance around the outside of a rectangular garden that is 8.375 meters by 6.284 meters.

77. In statistics, we often need to add up a bunch of numbers in a data set. Let’s practice. Add up the following numbers and find the sum.

 2.84 , 3.17 , 1.74 , 2.52 , 3.25 , 2.7 , 1.98 , 3

78. Let’s practice adding up a bunch of numbers. Add the following and find the sum.

 39.4 , 36.7 , 37.2 , 40.6 , 41.3 , 42 , 37.5 , 39

79. Greg went to the store to buy some groceries. The prices of the items he purchased (including tax) are listed below. What was the total amount spent at the store?

Apples : $4.89
Milk: $2.57
Bread: $1.94
Eggs: $3.08

80. The temperature before the storm was 22.487 degrees Celsius. After the storm the temperature had dropped to 13.529 degrees Celsius. How much did the temperature drop?

81. Lucia had $1849.22 in her checking account. She deposited a check from her work for $427.38 and then paid her mortgage of $1027.56 and her electricity bill of $72.83 . How much does Lucia now have in her checking account?

**Section 2B – Formulas with Exponents and Multiplying Decimals**

Look at the following formula.  This is a famous formula in geometry that gives the area of a circle with radius r. Remember that . Suppose we want to find the area of a circle with a radius of 1.5 cm. We will need to review multiplying decimals as well as be able to use exponents with decimals.

Let’s start with a review of multiplying decimals. We start by multiplying the numbers as if they were whole numbers. We then count the total number of decimal places to the right of the decimal in both of the numbers being multiplied. The answer will need the same total number of decimals places to the right of the decimal. For example let’s look at 3.5 x 0.81 . First we need to multiply 35 x 81. Remember to use partial products. 35 x 81 = 2835 . The question now is where to place the decimal? 3.5 has one place to the right of the decimal and 0.81 has two. So we have a total of 3 decimals to the right of the decimal. Hence our answer should also have 3 decimal places. So 2.835 is the answer. Notice that unlike addition and subtraction, we do not need to line up the place values when we multiply.

Occasionally we may want to multiply a decimal by a power of 10 (10, 100, 1000, 10000, etc.) When we multiply a decimal by one of these numbers the decimal point moves to the right the same number of places as the number of zeros in the numbers. For example 3.4 x 1000 , the decimal point will move 3 places to the right and we get 3400. Another example is 0.0000754 x 100000. Notice the decimal point will move 5 places to the right and we get 7.54 as our answer.

Try the following examples with your instructor.

Example 1: 0.036 x 1.49 Example 2: 1.2 x 0.0009

Example 3: 1.4562 x 100 Example 4: 1000 x 0.041

Recall that exponents are repeated multiplication. So  means 1.3 x 1.3 = 1.69 . Here is another example:  = 0.2 x 0.2 x 0.2 = 0.008 (answer).

Try the following examples with your instructor.

Example 5: Simplify  Example 6: Simplify 

Example 7: Let’s see if you are ready to find the area of a circle using the formula. Let’s see if we can find the area of a circle with a radius 1.5 cm. (Remember  and we should always do exponents before multiplying.)

**Practice Problems Section 2B**

For #1-33, simplify any exponents and then multiply.

1. 0.038 x 100 2. 1.42 x 1000 3. 218.3 x 10

4. 0.00073 x 10000 5. 4.56 x 100 6. 0.00226 x 1000

7. 1.29 x 1000 8. 0.000352 x 1000 9. 0.0358 x 100

10. 10 x 0.348 11. 10000 x 0.00057 12. 100 x 34.711

13. 1000 x 0.336 14. 100 x 0.9174 15. 0.3791 x 10

16.  (this is called a million and the prefix “mega” is used)

17.  (this is called a billion and the prefix “giga” is used)

18.  (this is called a trillion and the prefix “tera” is used)

19.  20.  21. 

22.  23.  24. 

25.  26.  27. 

28. 0.007 x 0.9 29. 0.012 x 0.07 30. 0.016 x 0.09

31. 8 x 1.75 32. 1.3 x 4.6 33. 0.007 x 0.24

34. 0.136 x 8.2 35. 0.006 x 0.017 36. 19.4 x 13.7

37. 7.4 x 0.31 38. 0.27 x 3.9 39. 41.8 x 0.006

40. 0.096 x 1.33 41. 12.7 x 0.044 42. 212.3 x 0.0082

43. 0.6 x 0.3 x 1.2 44. 0.043 x 10 x 0.7 45. 0.14 x 8 x 0.03

46.  47.  48. 

49.  50.  51. 

52.  53.  54. 

55.  56.  57. 

Use the formulato find the area of the following circles.
(Remember  and we should always do exponents before multiplying.)

58. Find the area of a circle with a radius of 3.2 feet.

59. Find the area of a circle with a radius of 1.4 mm.

60. Find the area of a circle with a radius of 7.5 inches.

61. Find the area of a circle with a radius of 2.9 cm.

A company makes a monthly profit of $2800.40. The total profit after m months is given by the formula T = 2800.4 x *m* .

62. Find the total profit after 12 months (1 year).

63. Find the total profit after 36 months (3 years).

Find the volume of cube with the formula  where V is the volume and *s* is the length of a side of the cube.

64. Find the volume of a cube if one side of the cube is 0.7 feet long.

65. Find the volume of a cube with if one side of the cube is 1.2 cm long.

The volume of a can (circular cylinder) is given by the formula  where r is the radius and h is the height. (Use )

66. Find out how much water a water tank can hold if the radius of the tank is 1.5 meters and the height is 10 meters. (Use )

67. What is the volume of a cylinder shaped trash can that has a radius of 2.1 feet and a height of 4 feet? (Use )

**Section 2C – Formulas with Dividing Decimals**

Look at the following z-score formula again from statistics.  . Suppose we want to calculate the z-score if . Not only do we need to be able to subtract decimals, but we also need to be able to divide decimals. So let’s review dividing decimals some.

Much like multiplying decimals, the key to dividing decimals is that we need to make sure to put the decimal in the correct place. We need to make sure that the number we are dividing by (divisor) is a whole number. If the divisor is a whole number then the decimal in the answer will be directly above where it is in the number we are dividing. That is pretty confusing, but I hope an example will help clear it up.

Look at  . One thing to note is that the first number (dividend) always goes on the inside of the long division. The number you are dividing by (divisor) goes on the outside.

 Since the divisor is not a whole number, we need to move the decimal one place to the right. We will have to move the decimal inside 1 place to the right as well. Hence the division becomes:

 Notice the dividend does not have to be a whole number. Only the divisor does. Now divide as if they are whole numbers. You may need to add zeros to the end of the dividend if it does not come out even.



Notice we keep adding zeros and dividing until we get a zero remainder or we see a repeating pattern. In this case we got a zero remainder after the thousandths place. Notice also the decimal point is directly above where it is in the dividend. This again is only the case when the divisor is a whole number. We can also use estimation to see if the decimal point is in the right place. 14 is slightly over half of 25, so the answer should be a little bigger than one-half (0.5).

Occasionally we divide decimals by powers of 10 (10, 100, 1000, etc.) Remember when you multiply by a power of 10 the decimal point simply moves to the right. It is similar with division. If you divide by a power of 10, you move the decimal point to the left the same number of places as the number of zeros in your power of 10. For example  moves the decimal point 3 places to the left so 0.00149 is the answer. A good way to remember the left and right rules is to think of it as bigger and smaller. When we multiply by a number like 1000, we are making the number larger, so the decimal point has to move to the right. When we divide by a number like 1000, we are making the number smaller, so the decimal point has to move to the left.

Let’s look at an example with a repeating pattern. Look at  . First we need to make the divisor a whole number. Remember the divisor is the number you are dividing by (0.99). Notice we will need to move the decimal point two places to the right to make 0.99 into a whole number (99). If we move the decimal point two places in divisor we also have to move the decimal point two places in the dividend (first number in the division). Moving the decimal point two places gives us 5.2 . Notice the dividend does not have to be a whole number. Once the divisor 0.99 becomes a whole number, the decimal point of the dividend will be in the correct place for our division. Note: Notice the process of moving the decimal is really the same as multiplying the numerator and denominator of a fraction by the same number (100). This is a common practice with fractions and always gives an equivalent fraction. So  is the same as . Be careful in the order of division. Remember the first number goes inside the long division and the second number goes outside. Notice that the remainders repeat, 52 then 25, then 52, then 25. This will go on infinitely. When this happens, we put a bar over the repeating part. So our answer is  . Notice that we do not need to move the decimal back. It is in the correct place. So  is the same as  . It is also good to note that we only need to put the bar over the first set of numbers that repeat. (For example, we do not need to write the answer as ).



Now you try some examples with your instructor.

Example 1:  Example 2: 

Example 3:  Example 4: 

Fractions can also be converted to decimals simply by dividing the numerator by the denominator. Remember the denominator is the divisor and has to go on the outside of the long division. For example the fraction 5/8 is really  , so if we add zeros to the 5 and divide, we get the following:

 Notice we did not have to move the decimal as our divisor 8 was already a whole number.

Note about zero and one: Remember any number (even a decimal) divided by 1, is itself. Any number divided by zero is undefined, and zero divided by any number (even a decimal) is zero.

Try some more examples with your instructor.

Example 5: Convert 3/16 into a decimal. Example 6: Convert 5/6 into a decimal. Place a bar over any repeating part.

Example 7: Divide:  Example 8: Divide: 

**Practice Problems Section 2C**

Divide the following and write your answer as a decimal or whole number.

1.  2.  3. 

4.  5.  6. 

7.  8.  9. 

10.  11.  12. 

13.  14.  15. 

16.  17.  18. 

19.  20.  21. 

22.  23.  24. 

25.  26.  27. 

28.  29.  30. 

31.  32.  33. 

34.  35.  36. 

37.  38.  39. 

40.  41.  42. 

43.  44.  45. 

Convert the following fractions into decimals. Put a bar over any repeating parts.

46. Convert  into a decimal. 47. Convert  into a decimal.

48. Convert  into a decimal 49. Convert  into a decimal.

50. Convert  into a decimal. 51. Convert  into a decimal.

52. Convert  into a decimal 53. Convert  into a decimal.

54. Convert  into a decimal. 55. Convert  into a decimal.

56. Convert  into a decimal 57. Convert  into a decimal.

58. Jim works in construction and gets paid $1847.30 every four weeks. How much does he make each week?

59. Miko needs $14623.40 for a down payment on a home. If she can save $860.20 each month, how many months will it take her to get the money she needs. (Not counting interest).

60. Elena owns a jewelry store and made $4865.77 in total sales by selling diamond bracelets. If each diamond bracelet costs $374.29 , then how many bracelets did she sell?

61. Wanda is a chemist working for a pharmaceutical company. She has a total of 880.46 mg of a new medicine. She needs to make pills where each pill has 7.5 mg in it. How many pills can she make? Will she have any left-over medicine?

Let’s see if we can solve the z-score problem from the beginning of the section. Use the formula 

62. Find the z-score if .

63. Find the z-score if 

The following formula calculates the volume of a cone in geometry. 

64. Find the volume of a cone with a radius of 4 cm and a height of 1.2 cm. (Use )

65. Find the volume of a cone with a radius of 0.5 yards and a height of 2 yards. (Use )

**Section 2D – Scientific Notation**

Numbers in science and statistics classes are sometimes very small (like the size of an atom) or very large (like the distance from Earth to the Sun). Numbers like that can have a huge amount of zeros and are very difficult to read, let alone interpret. Scientific Notation is a way of writing numbers that makes them more manageable, especially very small and very large quantities.

One number that is very important in statistics is “P-value”. P-value is often a number very close to zero. For example you may see a P-value = 0.000000000173 . This is a difficult number to interpret. Let’s talk about scientific notation and see if there is an easier way of writing this number.

To understand scientific notation we need to first review multiplying and dividing decimals by powers of ten. These concepts were covered in our last two sections. Notice a power of 10 like  is a 1 with four zeros or 10000. When we multiply a decimal by , we move the decimal 4 places to the right (making it larger). When we divide a decimal by , we move the decimal 4 places to the left (making it smaller).

Dividing a decimal by  is the same as multiplying by . The negative exponent means it is in the denominator of a fraction (division). Let’s review this idea. So if I multiply 3.5 x  = 3.5 x 10000 = 35000. If I multiply 3.5 x =  = 0.00035. Multiplying by a negative power of 10 makes the decimal small. Multiplying by a positive power of 10 makes the number large. OK, now we are ready for scientific notation.

Scientific Notation

A number is in scientific notation if it is a number between 1 and 10 , times a power of 10. For example we saw above that 35000 = 3.5 x . This is scientific notation. The first number is between 1 and 10 and we are multiplying by a power of 10. 35000 also equals 35 x  . This is not scientific notation since the first number is not between 1 and 10.

How do we convert a large or small number into scientific notation? Let’s look at an example. Suppose we are looking at a distance in astronomy of 83,000,000 miles. To write this number in scientific notation, ask yourself where the decimal point would be to make it a number between 1 and 10. Right now the decimal point is after the last zero. To be a number between 1 and 10 the decimal point needs to be between the 8 and 3. So our scientific notation number should be 8.3 x some power of 10. What power of 10 should we use? We moved the decimal point 7 places so the power of 10 should be  or , but which one? Do we need 8.3 to get larger or smaller so it will equal 83,000,000 miles? Larger, of course. So we should use the positive exponent since that makes numbers larger. So the scientific notation for 83,000,000 miles is 8.3 x .

Now you try the next couple of examples with your instructor.

Example 1: Write the number 0.0000049 Example 2: Write the number 63,500
 meters in scientific notation. pounds in scientific notation.

**Practice Problems Section 2D**

Convert the following numbers into scientific notation

 1. 2,450 2. 470,000 3. 13,600

 4. 0.00031 5. 0.0087 6. 0.00047

 7. 570,000 8. 36,000 9. 0.000000647

10. 7,400,000 11. 0.000132 12. 0.00004152

13. 5,200,000 14. 1,450,000 15. 0.0000089

16. 0.0003016 17. 0.00000132 18. 647,000,000

19. 52,600,000 20. 6,111,000,000 21. 0.0000993

22. 0.008144 23. 0.0000306 24. 189,000

25. 13,800 26. 6,458,000 27. 440,000

28. 0.0004881 29. 0.00000055 30. 0.0261

31. 246,670 32. 0.00000132 33. 0.0000011

34. 0.0000816 35. 672,100 36. 94,000

The following numbers are in scientific notation. Write the actual number they represent.

37. 1.48 x  38. 3.9 x  39. 4.975 x 

40. 6.42 x  41. 9.25 x  42. 7.113 x 

43. 7.6 x  44. 4.81 x  45. 2.59 x 

46. 8.94 x  47. 3.15 x  48. 1.45 x 

49. How can we tell if a number in scientific notation represents a large or small number?

50. Let’s see if you are ready to tackle the P-value example from the beginning of the section. Remember the P-value was 0.000000000173 . Write this number in scientific notation. We often compare P-values to 0.05 , so was this P-value larger or smaller than 0.05?

51. Statistics computer programs can also calculate P-values, but they are often given in scientific notation. Here is a printout from the stat program. (P-value = 3.578 x )

What number is this scientific notation representing? Is it larger or smaller than 0.05?

52. A science journal stated that the distance from the Earth to Pluto is approximately 4.67 billion (4,670,000,000) miles. Write this number in scientific notation.

53. An astronomy book stated that the distance between Mars and the Sun is approximately 2.279 x  kilometers. Since this is scientific notation, what is the actual number?

54. A chemistry book estimates that the diameter of a particular atom is about 0.0000000107 centimeter. Write this number in scientific notation.

55. We estimate that a human hair has a diameter of 0.00008 meters. Write this number in scientific notation.

56. A student said that the scientific notation for 6,800 feet is 68 x . Why is this not scientific notation?

57. A student said that the scientific notation for the diameter of a human hair is . Why is this not scientific notation?

**Section 2E – Significant Figure Rules**

When adding, subtracting, multiplying or dividing decimals it is important to know where to round the answer to. Most science classes base their rounding on something called significant figures. When taking science classes, it is vital to know the rules for significant figures.

Let’s start by defining what numbers in a decimal are significant and which are not. This will help us determine how many significant figures a number has.

Significant Figure Rules

* Non-zero numbers (1,2,3,4,…,9) are always significant. For example 2.5834 has five significant figures.
* Zeros between non-zero numbers are also considered significant. For example 20.0003 has six significant figures.
* Zeros at the end of a whole number are not considered significant. For example 31,000 has only two significant figures.
* Zeros to the right of a decimal point and before any non-zero numbers are not considered significant. For example 0.000027 has only two significant figures.
* In a decimal, zeros to the right of a non-zero number are significant. For example 0.3400 has four significant figures.

Significant figures and scientific notation relate well to one another. You will only use significant numbers in the scientific notation representation of the number. For example we said that 0.000027 has only two significant figures. Notice the scientific notation for this number only uses the two significant figures.

Now you try a couple of examples with your instructor.

Example 1: How many significant figures does the number 21.7934 have?

Example 2: How many significant figures does the number 0.004108 have?

Example 3: How many significant figures does the number 30,800,000 have?

Rounding Rules with significant figures
A. When adding or subtracting decimals, identify the number that has the least number of place values to the right of the decimal. Round the sum or difference to the same place value. For example  (Notice the 2.5 has only 1 decimal place to the right of the decimal. Since this was the least amount we rounded our answer to the same accuracy.)

B. When multiplying or dividing decimals, identify the number that has the least significant figures. Round your answer so that it has the same number of significant figures. For example  (notice that 12.831 has 5 significant figures and 17.4 has 3 significant figures. Since the least is 3, we will want our answer to have 3 significant figures. So we had to round to the ones place to have 3 significant figures.

Now you try a couple more examples with your instructor.

Example 4: Subtract the following, and use significant figure rounding rules to round the answer to the appropriate place.



Example 5: Divide the following, and use significant figure rounding rules to round the answer to the appropriate place.



Example 6: Multiply the following, and use significant figure rounding rules to round the answer to the appropriate place.

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**Practice Problems Section 2E**

Determine the number of significant figures for each of the following.

1. 7.569 2. 3.0062 3. 700,000

4. 8.200 5. 0.00051 6. 9,260,000

7. 358.1 8. 0.000572 9. 23.1

10. 90 11. 5.0174 12. 21,350

13. 33.7934 14. 1.00006 15. 28,700

16. 0.000309 17. 12,400 18. 1.3750

19. 0.000480 20. 35,481.2 21. 0.0003031

22. 55,600,000 23. 850 24. 84.007

25. 23.409 26. 200,000 27. 7.1190

28. 0.35 29. 0.000005 30. 0.000000403

Add or subtract the following. Then use significant figure rounding rules to round the answer to the appropriate place.

31.  32. 

33.  34. 

35.  36. 

37.  38. 

39.  40. 

41.  42. 

43.  44. 

45.  46. 

47.  48. 

Multiply or divide the following in order from left to right. Then use significant figure rounding rules to round the answer to the appropriate place.

49.  50. 

51.  52. 

53.  54. 

55.  56. 

57.  58. 

59.  60. 

61.  62. 

63.  64. 

**Section 2F – Estimating Square Roots and Order of Operations with Decimals**

Let’s look at the distance formula again. The distance between two points  and on a plane is  , but suppose the numbers we are plugging in are decimals. What do we do then? We already went over the order of operations. It will be very similar, but how do we estimate a square root when it is not a perfect square like 4 or 9?

Remember, the square root is the number we can square that gives us the number that is under the square root. For example,  means the number we can square (multiply by itself) to get 49. The answer is 7, of course, since  . But what happens when the number under the radical is not a perfect square? Perfect squares are 1,4,9,16,25,36,49,64,81,…

Let’s look at , for example. Can we approximate this radical? What we will do is try to find two numbers that 13 falls in between that we do know the square root of : like 9 and 16. Notice  , but  and  so that implies  . Hence  is a decimal between 3 and 4. Notice 13 is in the middle between 9 and 16, but it is slightly closer to 16 than it is to 9. So think of a decimal in the middle of 3 and 4 but slightly closer to 4. Let’s guess 3.6. Remember we are looking for a number that when we square it (multiply times itself), the answer is close to 13. Well, 3.6 x 3.6 = 12.96, which is very close to 13. Hence 3.6 is a pretty good approximation.

Note: You do not need to be perfect in your estimation. You just need to be close. For example, in the last problem you could approximate  as either 3.6 or 3.7, but 3.2 would be too far away since 3.2 x 3.2 = 10.24 (far from 13).

Do the following examples with your instructor.

Example 1: Estimate as a decimal Example 2: Estimate  as a decimal
 that ends in the tenths place. that ends in the tenths place.

Let’s review order of operations. The order of operations is the order we perform calculations. Remember, if you do a problem out of order, you will often get the wrong answer. Here is the order of operations.

1. Parenthesis  *(Start with the inner-most parenthesis first)*

2. Exponents and Square Roots

3. Multiplication and Division in order from left to right.

4. Addition and Subtraction in order from left to right.

For example, let’s look at  . We do parenthesis first, so 4.7-4.1 = 0.6 Now do the exponents and square roots, so  So now we have  Multiplication and division in order from left to right is next. Notice the division must be done before any addition or subtraction.  Adding the last two numbers gives us 0.0013 + 0.0036 = 0.0049 as our final answer.

Now do the following examples with your instructor.

Example 3: Simplify 

Example 4: Simplify 

**Practice Problems Section 2F**

Estimate the following square roots as a decimal that ends in the tenths place. Any approximation that is close is acceptable. Be sure to check your answer.

1.  2.  3. 

4.  5.  6. 

7.  8.  9. 

10.  11.  12. 

13.  14.  15. 

16.  17.  18. 

19.  20.  21. 

22.  23.  24. 

25.  26.  27. 

28.  29.  30. 

31.  32.  33. 

Perform the indicated operations for #16-21.

34.  35. 

36.  37. 

38.  39. 

40.  41. 

42.  43. 

44.  (Estimate with a decimal that ends in the tenths place)

Now let’s see if we are ready to tackle the distance formula with decimals. Estimate with a decimal that ends in the tenths place. 

45. Find the distance if 

46. Find the distance if 

47. Find the distance if 

A formula we referenced at the beginning of our book is the certificate of deposit
formula  .

48. Find the future value *A* of a certificate of deposit after 1 year *(t = 1)* if a person
invests $2000 *(P = $2000)* with an annual interest rate of 4% *(r = 0.04)* compounded twice a year *(n = 2)*.

49. Find the future value *A* of a certificate of deposit after 2 years *(t = 2)* if a person
invests $5000 *(P = $5000)* with an annual interest rate of 10% *(r = 0.1)* compounded once a year *(n = 1)*.

50. Find the future value *A* of a certificate of deposit after 1 year *(t = 1)* if a person
invests $10000 *(P = $10000)* with an annual interest rate of 6% *(r = 0.06)* compounded twice a year *(n = 2)*.

**Section 2G – Statistics Applications with Decimals**

Statistics is the science of collecting and analyzing data to learn about the world around us. Most scientific studies include statistical evidence. It is vital in today’s world to have some training in statistics. The word “statistic” refers to a number calculated from some sample data. In this section, we will be looking at several “statistics”. We will not be learning the meaning behind these calculations yet, but we will instead focus on how to calculate them correctly.

The first statistic we will learn to calculate the mean average. The mean average is found by adding up all the numbers in the data set and then dividing by how many numbers you have. Remember, the mean is a type of average for the data. However it is not the only type of average. The formula for the mean looks like this:  . In this formula the “*x”* refers to individual numbers in the data set, “*n”* refers to how many numbers are in the data set, and means to add.

Let’s look at an example of calculating the mean average.

Calculate the mean average weight of cars for the following data. The weights of the cars were measured in tons.

Car Data: 1.60 , 1.52 , 1.79 , 1.44



We often round statistics like the mean to one more decimal place to the right than is present in the original data. Notice all the numbers in the car data ended in the hundredths place. We will round our answer to one more decimal place to the right than the hundredths place. This means we should round the answer to the thousandths place.

Hence the mean average weight of the cars is approximately 1.588 tons.

The mean is not the only type of average. Another average that is very important in statistics is the median average. To find the median average, you must first put the data in order from smallest to largest. Now count the numbers to the middle. If there are an odd number of values in your data set, there will be a single number in the middle. That middle number is the median. If there are an even number of values in your data set, there will be two numbers in the middle. Add these two middle numbers and divide by 2.

Let’s look at the car weight data again. Remember all the numbers measure weight in tons. This time let’s find the median average.

Car Data: 1.60 , 1.52 , 1.79 , 1.44

First put the data in order from smallest to largest.

1.44 , 1.52 , 1.60 , 1.79

Notice if we count to the middle, there are two numbers in the middle (1.52 and 1.60). Add these two numbers and divide by 2 to get the median.



Hence the median average weight of the cars is 1.56 tons.

The next statistic we will learn to calculate is the overall range of the data. The range is not an average. It is a measure of how spread out the data is. To find the range, you must first identify the highest number in the data set (maximum value) and the lowest number in the data set (minimum value).

*Range = Max – Min*

For an example, let us look at the car data again. Identify the maximum and minimum values in the data and calculate the range.

Car Data: 1.60 , 1.52 , 1.79 , 1.44

Max = 1.79 tons and Min = 1.44 tons

*Range = Max – Min =* 1.79 – 1.44 = 0.35

So the range of the data is 0.35 tons.

Now do the following examples with your instructor.

Example 1: Julie is training to run a marathon. The following data shows how far Julie has ran this week in miles. Calculate Julie’s mean average, median average, and overall range for the running distances this week. Don’t forget to round the answer to one more decimal place to the right than the original data has.

10.7 , 15.3 , 13.9 , 22.5 , 8.3 , 14.1 , 23.3

Example 2: Harry is taking a finance class and has to keep track of how much he spends on lunch Monday through Saturday. (He didn’t eat out on Sunday.) His teacher wants him to calculate the mean average, the median average, and the range. Here is his data. Calculate the mean, median and range. Since this is money, round the answer to the nearest cent (hundredths place).

$7.84 , $9.61 , $6.32 , $11.79 , $8.46 , $14.28

**Practice Problems Section 2G**

*For each of the following data sets, calculate the mean average, median average, and the overall range. Remember to round the answers to one more decimal place to the right than the original data has. In the case of money, round to the nearest cent (hundredths place).*

1. Exam Scores: 92.6 , 73.1 , 69.7 , 87.4 , 94.5
2. Weights of herbal medicine in grams: 5.7 , 5.1 , 4.8 , 6.2 , 4.6 , 5.5 , 6.3 , 5.2
3. Price of movie ticket in dollars (from last ten years):
$8.58 , $8.43 , $8.17 , $8.13 , $7.96 , $7.93 , $7.89 , $7.50 , $7.18 , $6.88
4. Height of tree in feet over an eleven year period :
6.0 , 9.5 , 13.0 , 15.0 , 16.5 , 17.5 , 18.5 , 19.0 , 19.5 , 19.7 , 19.8
5. Number of milligrams of medicine given to patients at a hospital:
5.0 , 10.0 , 8.5 , 7.5 , 2.5 , 5.5 , 6.5 , 9.5 , 10.5 , 12.5 , 14.0 , 7.5 , 3.5 , 11.5
6. Amount of milk in cups from various chocolate cake recipes:
1.50 , 0.75 , 1.25 , 1.00 , 1.75 , 2.25 , 0.50
7. Area in square kilometers from various national parks in Alaska:
19185.8 , 30448.1 , 13044.6 , 14869.6 , 2711.3 , 7084.9 , 10601.6 , 33682.6
8. Amount of solar energy in kilowatt hours from solar program at a community college in Northern CA (six months of data, April through September) :
1437.1 , 1483.6 , 1324.6 , 1372.1 , 1348.1 , 1385.8
9. Amount of solar energy in kilowatt hours from solar program at a community college in Northern CA (six months of data, October through March) :
709.1 , 754.5 , 537.2 , 520.1 , 499.4 , 1236.5
10. The amount of money in a savings account in dollars:
$500.00 , $638.64 , $751.83 , $960.30
11. The exam scores on a philosophy test: 84.3 , 91.2 , 94.6 , 81.3
12. The number of tons of trash brought by trucks to a trash dump:
 5.7 , 3.2 , 7.6 , 9.1 , 8.2 , 6.9
13. The cost of various laptop computers: $866.23 , $1524.90 , $582.35 , $ 799.63
14. Height of women on a basketball team in inches: 72.1 , 69.5 , 75.2 , 77.3 , 65.8
15. Weight of men on a professional football team in pounds:
 255.5 , 310.4 , 269.0 , 247.5 , 330.2 , 239.7
16. Time (in minutes) to run a mile for athletes on a cross country track team:
 5.25 , 4.85 , 4.30 , 5.05 , 4.52
17. Price of various bass guitars in dollars:
 $399.99 , $799.50 , $849.99 , $2325.00 , $529.60 , $1850.49
18. Length of various two-by-fours at a lumber yard in meters.

 1.75 , 1.50 , 1.35 , 2.25 , 2.75 , 0.80 , 1.25

1. Prices in cents of a pound of Columbian coffee during various months:
 82.07 , 91.55 , 103.24 , 101.79 , 99.14 , 96.01 , 92.45
2. The average temperature in Fargo, North Dakota during winter in degrees fahrenheit:

9.1 , 12.6 , 0.4 , 3.1 , 7.4 , 18.2 , 5.4 , 1.7 , 6.9

**Chapter 2 Review**

In Module II, we reviewed how to do calculations with decimals. We looked at the importance of place value in addition and subtraction. We looked at how to determine where the decimal point should go in multiplication and division problems. We learned how to write a number in scientific notation and estimate square roots and we reviewed the order of operations. With these tools, we looked at some real formula applications and calculated the mean, median and range.

Here are some key things to remember from chapter 2.

* It is vital to memorize the place values. For example the first number to the left of the decimal is the ones place, the first number to the right of the decimal point is the tenths place, and the second number to the right of the decimal place is the hundredths place…
* To round decimals to a certain place value, look to the number to the right of the place value. If it is 5 or above, round up (add 1 to place value, cut off the rest of decimal). If it is 5 or above, round up (add 1 to place value, cut off the rest of decimal). If it is 4 or less, round down (leave place value alone, cut off the rest of decimal).
* Non-zero numbers are always significant. Zero’s between two non-zero numbers are also significant. Zeros at the end of a whole number are not significant. Zeros at the beginning of a decimal are not significant.
* To add or subtract decimals, you have to line up the decimals and place values. Keep the decimal in the same place in the answer.
* To multiply decimals, multiply the numbers as if the decimals were not there. Count the number of places to the right of the decimal point in the first number. Count the number of places to the right of the decimal point in the second number. Add the total number of places to the right for both numbers. The answer will need the same number of values to the right of its decimal place. For example, if the first number in the product has 3 places to the right and the second number has 2 places then your answer needs 3+2=5 numbers to the right of the decimal point.
* To divide decimals, make sure the number you are dividing (dividend) goes on the inside of the long division and the number you are dividing by (divisor) goes on the outside. Remember the devisor must be whole number. Move the decimal in the divisor to the right until the divisor is a whole number. Move the decimal in the dividend the same number of places. When the divisor is a whole number, the decimal in the dividend is the same as your answer. Just bring the decimal up from the dividend.
* Scientific Notation means a number between 1 and 10 times a power of 10. For example  . Small numbers have a negative power of 10 and large numbers have a positive power of 10.
* To find the mean average, add up all the numbers and divide by how many numbers you have. To find the median average, put the numbers in order from smallest to largest. If there is 1 number in the middle, then that is the median. If there are two numbers in the middle, the median is half way in between the two numbers in the middle. (Add them and divide by 2.) The range is the largest number in the data minus the smallest number in the data.

**Chapter 2 Review Problems**

Perform the indicated operation for #1-40.

1. 2.3 + 0.14 + 0.575 2. 1.56 + 19.8 + 317 3. 0.478 + 9.69

4. 89.61 + 4.530 5. 0.875 + 64.9 6. 90.52 + 72.8

7. 20 – 8.31 8. 3 – 1.472 9. 0.15 – 0.0854

10. 53.1 – 0.175 11. 1.348 – 0.936 12. 4.421 – 3.89

13. 6 x 1.9 14. 0.7 x 0.42 15. 2.43 x 50

16.  17. 1000 x 0.084 18. 7.4 x 10000

19. 6.5 x 7.2 20. 0.84 x 0.89 21. 0.24 x 1.48
22. 7.5 x 3.48 23. 13.5 x 0.64 24. 307 x 0.52
25. 1.94 x 0 26. 2.34 x 1 27. 0 x 0.138
28. 1 x 23.45 29.  30. 
31.  32.  33. 
34.  35.  36. 
37.  38.  39. 
40. 

41. Which number is larger, 0.07 or 0.0699?

42. Which number is smaller, 1.358 or 1.5?

43. Round 1.3856 to the tenths place.

44. Round 0.024879 to the hundredths place.

45. Round 1.379684 to the thousandths place.

46. Write the number 530,000,000 in scientific notation.

47. Write the number 0.00000073 in scientific notation.

48. What number is represented by 4.2 x  ?

49. What number is represented by 1.16 x  ?

50: How many significant figures does the number 21.7 have?

51: How many significant figures does the number 1.00032 have?

52: How many significant figures does the number 22,900,000 have?

For #53-56, perform the operations and use significant figure rounding rules to round the answer to the appropriate place.

53. 

54. 

55. 

56. 

Perform the indicated operations for #57-61. Estimate any square roots with a decimal that ends in the tenth’s place.

57.  58.  59. 

60.  61. 

The surface area of a box is given by the formula  .

62. Find the surface area of a box if *L = 8.3cm, W = 6.3 cm, and H = 5.7 cm.*

63. Find the surface area of a box if *L = 2.4 in, W = 1.6 in, and H = 1.2 in.*

Let’s look at the z-score formula again. 

64. Find the z-score if  .

65. Find the z-score if 

Use the certificate of deposit formula  .

66. Find the future value *A* of a certificate of deposit after 2 years *(t = 2)* if a person
invests $6000 *(P = $6000)* with an annual interest rate of 4% *(r = 0.04)* compounded once a year *(n = 1)*.

67. Find the future value *A* of a certificate of deposit after 1 year *(t = 1)* if a person
invests $2000 *(P = $2000)* with an annual interest rate of 7% *(r = 0.07)* compounded twice a year *(n = 2)*.

Mean, Median, and Range problems

68. The largest living lizard is the Komodo dragon. We measured the lengths of these lizards in meters. Calculate the mean average, median average and the range of the Komodo dragon data. Round the answers to one more decimal place to the right than the data has.

Komodo dragon lengths data: 2.7 , 3.3 , 2.6 , 2.5 , 2.9 , 3.4 , 3.1

69. Gas prices in Los Angeles fluctuate a lot. We took a sample of various gas stations around Los Angeles and recorded the price per gallon in dollars. Calculate the mean average, median average and the range of the gas price data. Round the answers to the nearest cent (hundredths place).

Gas Price Data:
$3.69 , $3.57 , $3.49 , $3.37 , $3.19 , $3.15 , $3.09 , $2.99 , $2.83 , $2.65