

Chapter 4 – Formulas and Negative Numbers

Section 4A – Negative Quantities and Absolute Value

Introduction: Negative numbers are very useful in our world today. In the stock market, a loss of \$2400 can be represented by a negative value. If you live in cold climates, you may have seen the temperature dip below zero. If the unemployment rate decreases over a given time period, this can be represented as a negative slope. If you ever visited Death Valley, you would know that its altitude is below sea level. There are many other uses for negative numbers. Many functions and formulas stem from a good understanding of negative numbers.

Let's look at a few examples. Suppose the temperature is 8 degrees below zero. Is this a positive quantity or a negative quantity?

Temperatures above zero are positive, while temperatures below zero are negative. So a temperature of 8 degrees below zero is -8 .

Suppose we lost \$375 on our trip to Las Vegas. Is this a positive quantity or a negative quantity?

A financial gain is positive, while a loss is negative. So our loss of \$375 in Las Vegas corresponds to -375 .

Let's look at an example. Suppose a mountain range is 8,400 feet above sea level. Is this a positive quantity or a negative quantity?

Above sea level is positive, while heights below sea level are negative. So a mountain at 8,400 feet above sea level corresponds to $+8,400$ or just 8,400. (You do not have to write the positive sign, though in this section it is helpful.)

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Another useful topic is absolute value. The absolute value of a number is its distance from zero. So $|-16|$ is asking how far -16 is from zero on the number line. Since -16 is 16 places from zero, we know that $|-16| = +16$. Similarly we see that $|+10| = +10$ since $+10$ is 10 places from zero. It is not surprising that an absolute value will always be equal to a positive quantity.

Do the following examples with your instructor:

Example 1: Describe the following as a positive or negative quantity.

“The temperature is now 20 degrees below zero.”

“We made \$2500 in profit from our stocks.”

Example 2: Find the $|-9|$

Example 3: Find the $|+7|$

Practice Problems Section 4A

Describe the following as a negative or positive quantity.

1. The depth of a submarine is 347 feet below sea level.
2. The temperature in Juno Alaska is 13 degrees below zero.
3. An airplane is flying at 600 feet above sea level.
4. Mario lost \$650 while gambling at a Casino.
5. The amount of people in a small town has increased by 850.
6. A whale is swimming 120 feet below the surface of the ocean.

7. The temperature in Montana is 19 degrees below zero.
8. An airplane is flying at an altitude of 200 meters above sea level.
9. Niki won \$73 while playing poker with her friends.
10. The amount of people in a small town has decreased by 124.
11. A stock price decreased \$328 per share.
12. Sierra lost \$1,450 while gambling at a Casino.
13. The number of cases of the flu have increased by about 3500.

Find the following absolute values

- | | | |
|-----------------------|-----------------------|-----------------------|
| 14. $ -7 $ | 15. $ -12 $ | 16. $ +17 $ |
| 17. $ +18 $ | 18. $ -23 $ | 19. $ +16 $ |
| 20. $ -8.5 $ | 21. $ +7.44 $ | 22. $ -2.9 $ |
| 23. $ -19 $ | 24. $ -32 $ | 25. $ +49 $ |
| 26. $ +5\frac{1}{4} $ | 27. $ -7\frac{2}{5} $ | 28. $ +9\frac{1}{8} $ |
| 29. $ -5.7 $ | 30. $ +2\frac{3}{4} $ | 31. $ +5.913 $ |
| 32. $ -22 $ | 33. $ +3.5 $ | 34. $ -6.25 $ |

Section 4B – Adding Negative Quantities

Sally invested in a stock. When looking at the stock's price per share over that last few weeks, she noticed that the stock price had decreased \$4.50, then increased \$1.25 and lastly decreased again \$5. How much has the stock price fallen overall? These are the types of problems we will be looking at in this section. When the stock decreases, that corresponds to a negative value. When the stock increases, that corresponds to a positive value.

How do we add these quantities? Let's review.

To add two numbers with the same sign, (both negative or both positive) we add the numbers and keep the sign. For example, what is $-10 + -7$? Since both the numbers are negative, we simply add $10 + 7 = 17$ and then keep the negative sign. So $-10 + -7 = -17$.

To add two numbers with opposite signs (one negative and one positive) we subtract the numbers and keep the sign of the larger. For example, what is $-13 + +8$? Since the signs are opposite, we subtract $13 - 8 = 5$ and keep the sign of the larger. 13 is larger than 8 and the sign on the 13 is negative, so our answer will be $-13 + +8 = -5$.

Money is a great example of negative numbers. Suppose you owe the bank \$240 for your car payment. Notice this would correspond to (-240) . If you were only able to pay them \$180 $(+180)$, would you still owe the bank money? If so how much? Since we still owe the bank \$60, (-60) this would show that $-240 + +180 = -60$.

Add the following numbers with your instructor.

Example 1: $-14 + +11$

Example 2: $-12 + -6$

Example 3: $-17 + +4 + -15$

The same rules of addition apply to fractions and decimals. When we add two fractions or decimals with the same sign, we add the numbers and keep the sign. When we add two fractions or decimals with opposite signs we subtract the numbers and keep the sign of the larger.

Add the following numbers with your instructor.

Example 4: $-7.325 + +2.89$

Example 5: $-\frac{5}{6} + \left(-\frac{1}{4}\right)$

Practice Problems Section 4B

Add the following:

1. $-7 + -12$

2. $+9 + -15$

3. $-14 + -8$

4. $-9 + +7$

5. $-12 + -3$

6. $-17 + +17$

7. $-12 + -16$

8. $+8 + -33$

9. $-13 + -43$

10. $-19 + +4$

11. $-26 + -14$

12. $+57 + -24$

13. $-35 + -27$

14. $+28 + -53$

15. $-23 + -48$

16. $-109 + +109$

17. $-76 + -59$

18. $-538 + -249$

Remember the same rules of addition apply to fractions and decimals. Add the following.

19. $-2.6 + -5.71$

20. $+7.78 + -9.52$

21. $-0.68 + -0.349$

22. $+3.5 + -12.89$

23. $-7.31 + -9$

24. $-0.0047 + +0.185$

25. $\left(-\frac{1}{3}\right) + \left(-\frac{3}{4}\right)$

26. $\frac{6}{7} + \left(-\frac{1}{5}\right)$

27. $\left(-\frac{5}{8}\right) + \left(-\frac{7}{12}\right)$

28. $\left(-1\frac{1}{3}\right) + \left(+4\frac{3}{4}\right)$

29. $\left(-8\frac{1}{4}\right) + \left(-3\frac{5}{7}\right)$

30. $\left(+1\frac{3}{8}\right) + \left(-2\frac{5}{6}\right)$

There are many applications of negative numbers. Try these.

31. Jim gambles on fantasy football. On week 1, Jim lost \$23. On week 2, he lost \$14. On week 3, he made \$8. On week 4, he lost \$33. What was his net gain or loss?

32. The temperature on Monday was $+3^{\circ}F$. On Tuesday the temperature dropped 7 degrees and on Wednesday it dropped 5 more degrees. What was the temperature on Wednesday?

33. Jesse was scuba diving at a depth of 65 feet below sea level. He then dived an additional 19 feet down. He then swam up 23 feet. What was his new depth?

34. In Reno, NV, the temperature at 5:00 AM was $-8^{\circ}F$. Over the next 7 hours, the temperature had risen $29^{\circ}F$. What was the new temperature at noon?

35. A roadside fruit stand has a credit balance of -543 dollars. In the summer, they make a profit of \$6,150. What is their new balance?

36. A basketball player's "plus/minus" is a number that is calculated by adding all of the positive points her team scores with all of the negative points her team gives up while she is in the game. While the starting point guard was in the game, her team scored 62 points and gave up 71 points. Calculate her plus/minus value.

37. Paloma balancing her checkbook for the month of May. At the beginning of the month, she had a balance of \$89.05 in her checking account. She wrote a check for \$450, took out \$40 from the ATM, and deposited a paycheck worth \$915.25. What is Paloma's new account balance?

Section 4C – Subtracting Negative Quantities

Suppose we want to find the difference between 7 degrees below zero and 16 degrees below zero. Remember to find the difference between quantities we need to subtract, but how do we subtract negative numbers? That is the topic of this section.

The key to subtracting signed numbers is the idea of an opposite. Opposites have the same number, but have the opposite sign. For example the opposite of -4 is +4. The opposite of +7.241 is -7.241.

Look at the following key example. Suppose Julie has \$24 and then spends \$5, how much will she have left? We know from previous experience that this is a take away and can be found by the subtraction $24 - 5 = \$19$. Notice that losing \$5 can also be thought of as adding -5. In fact if we added -5, we get the following. $+24 + ^-5 = +19$. Did you notice that we get the same answer? This shows that $24 - 5 = +24 - ^+5 = +24 + ^-5$. Notice subtracting a quantity is the same as adding the opposite of the quantity. Subtracting +5 is the same as adding the opposite of +5. Since the opposite of +5 is -5, we see that subtracting +5 is the same as adding -5. This principle of adding the opposite can be applied to any subtraction problem. We simply add the opposite.

For example, suppose we want to subtract the following. $+7 - ^-4$. Again we can change the subtraction into adding the opposite. Remember that we want to take the opposite of the number being subtracted. This is the second number. Do not take the opposite of the first number. The opposite of -4 is +4. So subtracting -4 is the same as adding +4. Hence

$+7 - ^-4 = +7 + ^+4 = +11$. Notice the +7 stayed the same.

Let's look at another example. $^-9 - ^-10$. Again the opposite of -10 is +10 so we get $^-9 - ^-10 = ^-9 + ^+10$. Now we need to remember our addition rules for negative numbers. To add two numbers with opposite signs, we subtract the numbers and keep the sign of the larger. Hence $10 - 9 = 1$. Since the 10 was positive, our answer will also be positive (+1).

What happens when we add opposites? Look at $^-13 + ^+13$. To add two numbers with opposite signs, we subtract the numbers and keep the sign of the larger. So $13 - 13 = 0$. 0 is the only number that is neither positive nor negative. So $^-13 + ^+13 = 0$. Hence when we add two opposites, the answer is always zero.

Subtract the following examples with your instructor.

Example 1: $-4 - +20$

Example 2: $-9 - -6$

Example 3: $-8 - -8$

Example 4: $-0.29 - +0.885$

Example 5: $\frac{1}{4} - \frac{5}{6}$

Practice Problems Section 4C

Subtract the following.

1. $+3 - +9$

2. $-12 - +6$

3. $+8 - +7$

4. $+3 - -10$

5. $-9 - -3$

6. $+8 - -12$

7. $-14 - -5$

8. $-17 - +6$

9. $-8 - -8$

10. $+21 - +30$

11. $-16 - +11$

12. $+7 - +7$

13. $-14 - +6$

14. $-17 - -12$

15. $+18 - +11$

16. $+13 - -16$

17. $-17 - -21$

18. $-28 - -19$

19. $-24 - +16$

20. $-52 - +36$

21. $-26 - -26$

22. $+19 - +34$

23. $-23 - +14$

24. $+19 - +19$

Remember that the idea of adding the opposite also applies to fractions and decimals. Try the following.

25. $+0.074 - +0.183$

26. $-\frac{2}{3} - \left(+\frac{1}{6}\right)$

27. $+5.7 - +2.61$

28. $+\frac{1}{8} - \left(-\frac{1}{5}\right)$

29. $-9.17 - -6.8$

30. $\frac{3}{7} - 2\frac{5}{7}$

There are many applications of subtraction.

31. One scuba diver is 24 feet below sea level. A second scuba diver is 37 feet below sea level. How much deeper is the second scuba diver than the first?

32. Death Valley has a height of 86 meters below sea level. Mount Everest has a height of 8,848 meters above sea level. How much taller is Mount Everest than Death Valley?

33. Mark and Ryan both gamble on horse races. One Sunday, Mark lost \$235 and Ryan lost \$197. How much more did Mark lose than Ryan?

34. Aria owes the electric company \$73 for June and owes \$112 for July. What is the difference between the two bills?

35. The daytime temperature on the moon's surface can reach 253 degrees Fahrenheit, while the nighttime temperature can dip to 243 degrees Fahrenheit below zero. What is the difference between these two temperatures?

36. On a winter day in Dallas, TX, the projected high temperature during the day is 12 degrees Celsius, which is 21 degrees higher than the projected low temperature that night. What is the projected low temperature?

37. A small business that sells hats in a mall kiosk has a credit balance of $-11,239$ dollars. If in the next 2 months they lose \$2,460, what will their new balance be?

38. A basketball player's "plus/minus" is a number that is calculated by adding all of the positive points her team scores with all of the negative points her team gives up while she is in the game. While the starting center was in the game, her team scored 45 points and her plus/minus value was -8 . Subtract 45 from -8 to determine how many points her team gave up while she was in the game.

Section 4D – Multiplying and Dividing Negative Quantities

Every month, Sophie pays her rent and her bank account balance decreases \$457. If she pays her rent six months in advance, how much did her account decrease? Since a loss of \$457 can be thought of as -\$457, Sophie has -457 six times or $+6 \times -457$, but how do we multiply and divide negative numbers? Let's review.

If we think about the example above, Sophie loses \$457 six times so she has to have lost a total of $6 \times 457 = 2742$. Hence her account balance decreased \$2742. This gives us insight into the first rule for multiplication. The example shows that $+6 \times -457 = -2742$. When we multiply or divide two numbers with opposite signs the answer is always negative.

For example, look at $-8 \times +14$. Since they have opposite signs, we multiply the numbers. The answer will be negative. Hence $-8 \times +14 = -112$.

Note: This is not true for addition. When we add two numbers with opposite signs, the answer can be negative or positive depending on which is larger. For example: $-8 + +14 = +6$ Don't confuse addition rules with multiplication.

To multiply or divide two numbers with the same sign, we multiply or divide and the answer will always be positive.

For example, look at -9×-7 . Since they have the same sign (both negative), we multiply the numbers ($7 \times 9 = 63$). The answer will be positive. Hence $-9 \times -7 = +63$.

Note: You will hear people say "two negatives makes a positive". This is a bad thing to say and is not always true. This is especially not true for addition. When we add two numbers each with a negative sign, the answer will always be negative. For example: $-8 + -4 = -12$ Don't confuse addition rules with multiplication!

These rules apply to division as well. Look at $-18 \div -3$. Since this problem involves division with the same sign the answer will be positive. Hence $-18 \div -3 = +6$. Another example is $+32 \div -8$. Since this problem involves division with opposite signs the answer will be negative.

Hence $+32 \div -8 = -4$.

Note: Remember any number times 0 will equal 0. Also $0 \div$ any number will equal 0. Also we cannot divide by zero, so any number $\div 0$ is undefined. These apply to negative numbers as well.

Do the following examples with your instructor:

Example 1: -9×-13

Example 2: $-15 \times +3$

Example 3: $(-8)(-10)$ (Parenthesis together also means multiply)

Example 4: 0×-37

Example 5: $-17 \times +1$

Example 6: $-110 \div +11$

Example 7: $\frac{-29}{0}$

Practice Problems Section 4D

For #1-30, perform the following operations.

1. $-7 \times +2$

2. -5×-13

3. $-12 \times +8$

4. $(-16)(+3)$

5. $(+9)(-11)$

6. $(-21)(-2)$

7. $-15 \div +3$

8. $-35 \div -5$

9. $0 \div -1$

10. -45×0

11. $-51 \div +17$

12. $+85 \div -17$

13. $-8 \times +12$

14. $\frac{-13}{0}$

15. $\frac{-36}{-4}$

16. $-9 \times +15$

17. -12×-20

18. $-14 \times +6$

19. $(-25)(+7)$

20. $(+7)(-23)$

21. $(-42)(-10)$

22. $-75 \div +15$

23. $-144 \div -9$

24. $0 \div -13$

25. -19×0

26. $-64 \div +16$

27. $+1233 \div -9$

28. $-76 \times +1$

29. $\frac{-305}{0}$

30. $\frac{-1696}{-8}$

As with all the rules in this chapter, they apply to decimals and fractions as well.

31. $+3.4 \times -1.8$

32. $\left(-\frac{9}{16}\right)\left(\frac{8}{27}\right)$

33. $(-0.15)(-0.43)$

34. $-3.42 \div +0.3$

35. $\left(-\frac{5}{14}\right) \div \left(\frac{15}{7}\right)$

36. $-0.042 \div -0.25$

There are many applications involving multiplication and division with negative numbers. Look at the following.

37. Juan pays \$45 a month for his electric bill. His bank statement indicates that Juan has paid a total of \$675 to the electric company. How many months has he paid this bill?

38. The temperature Monday morning was $-3^{\circ}F$. During the day the temperature dropped 4 degrees six times. What was the temperature Monday night?

39. Charlie is building a new house that will not be connected to any city water pipes, so he plans to dig a well that will reach -100 ft deep. If he can dig -8 ft each hour, how many hours will it take him to reach his goal of -100 ft?

40. A football team lost 6 yards on each of the first 3 rushing (running) plays. What was the team's rushing total?

41. An investment banker was fired after losing \$600,000 over his first year on the job. On average, how much did he net each month?

A common formula in statistics is $x = \mu + (z)(\sigma)$.

42. If $\mu = 8.24$, $z = -1.35$ and $\sigma = 3.61$, find x . (Remember to do multiplication before addition.)

43. If $\mu = 62.9$, $z = -3.48$ and $\sigma = 2.8$, find x . (Remember to do multiplication before addition.)

44. If $\mu = -15$, $z = +2.4$ and $\sigma = +1.75$, find x . (Remember to do multiplication before addition.)

Section 4E – Exponents and Order of Operations with Negative Quantities

A common formula in Chemistry is the formula to convert degrees Fahrenheit into degrees

Celsius. $C = \frac{(F - 32)}{1.8}$. Since temperature often involves negative quantities, we see that we need to make sure that we can use the order of operations correctly.

The main key is to remember that the order of operations does not change just because some or all of the numbers are negative. We still do parenthesis first, then exponents, then multiplication or division in order from left to right, and last addition and subtraction from left to right.

Exponents can be particularly tricky with negative numbers. We need to make sure that we keep in mind whether or not the number is in parenthesis or not. For example:

For $(-7)^2$, notice that the -7 is in parenthesis, so the negative is applied first, then the square.

Hence $(-7)^2 = -7 \times -7 = +49$.

But what happens when there is no parenthesis?

If we see -7^2 , this is saying subtract 7 squared. By order of operations we do the square first, then subtract. Hence we see that $7^2 = 7 \times 7 = 49$, then we subtract. Hence $-7^2 = -49$

Try the following problems with your instructor.

Example 1: $(-3)^4$

Example 2: -3^4

Example 3: $-6 - (+14) \div (-2)(-3) + (-12)$

Example 4: Use the temperature formula $C = \frac{(F - 32)}{1.8}$ to convert $-7.6^\circ F$ into Celsius.

Practice Problems Section 4E

Simplify the following.

1. -9^2

2. -2^4

3. $(-12)^2$

4. $(-2)^4$

5. $(-6)^3$

6. -20^2

7. -2^3

8. $(-10)^2$

9. -8^2

10. -1^5

11. $(-7)^3$

12. $(-13)^2$

13. $(-3)^4$

14. -4^3

15. $(-5)^3$

16. $(-10)^4$

17. -15^2

18. $-(-6)^2$

19. $-7 + -4 \times -3$

20. $(-2)^4 - 8^2$

21. $(-12 - +9) \div (-7)$

22. $(-15)^2 - 4^2$

23. $-7(+8) \div (-2 \times +4) + (-12)$

24. $(-8 - -7)^3 + -24 \div +3$

25. $-6 \left[(+13 + -14)^3 \times (-7) \right]$

26. $(-13 + -7)^2 \div (-100)$

Use the chemistry formula $F = 1.8 \times C + 32$ to convert Celsius into Fahrenheit.

27. Convert $-4^\circ C$ into degrees Fahrenheit.

28. Convert $-13^\circ C$ into degrees Fahrenheit.

Use the chemistry formula $C = \frac{(F - 32)}{1.8}$ to convert Fahrenheit into Celsius.

29. Convert $26.6^{\circ}F$ into degrees Celsius.

30. Convert $-2.2^{\circ}F$ into degrees Celsius.

(For #31-33) In statistics, we use regression formulas to help businesses make predictions about things like profits and costs. A company makes flat screen TVs. We used regression to find the formula $P = -12.8(x - 43)^2 + 19,780$ where x represents the number of TVs made per day and P represents the profit in dollars.

31. If a company makes 23 TVs in a day, how much profit will they expect to make?

32. If the equipment malfunctions and the company cannot make any TVs on one day, how much money will we expect them to lose?

33. We calculated that the company should strive to make 43 TVs per day, as that value maximizes their profit. What would their maximum profit be if they made exactly 43 TVs in a day?

34. A small business starts out with a net loss of $-1,200$ dollars per month over the first 4 months in operation, and follows that with a net profit of $+800$ dollars per month over the next 8 months. How much money did the business make or lose in its first year?

35. A football running back loses 4 yards per carry for 5 of his rushing attempts. He also gains 8 yards per carry for 2 of his attempts. What are his total yards gained or lost?

Chapter 4 Review

A good understanding of negative numbers is vital for many applications. Finance, business, temperature and altitude are a few examples that frequently use negative numbers. Here are a few key things to remember.

- When adding two negative numbers, add the numbers and keep the sign.
- When adding a negative and positive number, subtract the numbers and keep the sign of the larger.
- When subtracting, we can rewrite any subtraction as adding the opposite. Then we can use the addition rules to finish the problem.
- When multiplying or dividing two negative numbers, the answer will always be positive.
- When multiplying or dividing a negative number and a positive number, the answer will always be negative.
- Don't confuse the addition and multiplication rules. (Two negatives does not always make a positive!)

Review Problems Chapter 4

Describe the following as a negative or positive quantity.

1. The depth of a scuba diver is 53 feet below sea level.
2. The temperature is 16 degrees below zero.
3. An airplane is flying at 2500 feet above sea level.
4. Pat won \$250 while gambling at a Casino.
5. The amount of people in a small town has decreased by 87.

Find the following absolute values:

6. $|-11|$

7. $|+24|$

8. $|-19|$

Add the following:

9. $-38 + -17$

10. $+8.2 + -13.7$

11. $-7\frac{1}{6} + -1\frac{5}{6}$

12. $-124 + +527$

13. $-0.0014 + -2.579$

14. $\frac{1}{12} + \left(-\frac{5}{6}\right)$

Subtract the following:

15. $-21 - +9$

16. $+17 - -6$

17. $-3 - -3$

18. $+21 - +30$

19. $+8.4 - +25.7$

20.

$-\frac{5}{8} - \left(\frac{3}{4}\right)$

Multiply the following:

21. $-9 \times +12$

22. -12×-3

23. $-2.6 \times +1.8$

24. $(-16)(+1)$

25. $(+0.09)(-1.1)$

26. $(-21)(0)$

Divide the following:

27. $0 \div (-8)$

28. $-0.48 \div +0.9$

29. $\left(\frac{1}{2}\right) \div \left(-\frac{5}{7}\right)$

30. $-2.39 \div -1$

31. $\frac{-15}{0}$

32. $\frac{-2.73}{-3}$

Simplify the following:

33. -13^2

34. $(-4)^2$

35. $-5^2 - (-3)^2$

36. $-2(-18) \div (-1 \times +9) + (-8)$

Answer the following application questions:

37. On Monday, John made \$234 in the stock market. On Tuesday, he lost \$417 and on Wednesday he lost an additional \$175. What was his net gain or loss over the three-day period?

38. Use the temperature formula $F = 1.8 \times C + 32$ to convert $-6^\circ C$ into Fahrenheit.

39. Use the temperature formula $C = \frac{(F - 32)}{1.8}$ to convert $28.4^\circ F$ into Celsius.

40. We used Statistics to find the following Cost formula for a company.

$C = 1.05(x - 33)^2 + 1240$ (x represents the number of hours their employees work per week and C represents the company's cost in dollars.) Find the company's cost if the employees work 40 hours in a week. Find the company's cost if the employees work 35 hours in a week.