Chapter 5 – Simplifying Formulas and Solving Equations

Look at the geometry formula for Perimeter of a rectangle P = L + W + L + W. Can this formula be written in a simpler way? If it is true, that we can simplify formulas, it can save us a lot of work and make problems easier. How do you simplify a formula?

A famous formula in statistics is the z-score formula $z = \frac{x-\mu}{\sigma}$. But what if we need to find the x-value for a z-score of -2.4? Can we back solve the formula and figure out what x needs to be?

These are questions we will attempt to answer in chapter 5. We will focus on simplifying expressions and solving equations.

Section 5A – Simplifying Formulas and Like Terms

The key to simplifying formulas, is to understand "Terms". A "term" is a product of numbers and or letters. A term can be a number by itself, a letter by itself, or a product of letters and numbers. Here are some examples of terms:

12b

-11xy

W

-4

$7x^2$

As you can see, a term has two parts: a numerical coefficient (number part) and most of the time a variable part (letter). Let's see if we can separate these terms into their numerical coefficients and variable part.

12b : We see that 12 is the numerical coefficient and b is the variable (letter) part

-11xy: We see that -11 is the numerical coefficient and xy is the variable (letter) part

(This chapter is from Preparing for Algebra and Statistics , Second Edition by M. Teachout, College of the Canyons, Santa Clarita, CA, USA)

W: This is an interesting term as we don't see a number part. There is a number part though. Since W = 1W, We see that 1 is the numerical coefficient and of course W is the variable (letter) part.

-4: This is also an interesting case as there is no variable part. This is a special term called a constant or constant term. Constants have a number part (-7) but no variable part.

$7x^2$: We see that 7 is the numerical coefficient and x^2 is the variable (letter) part.

The degree of a term is the exponent on the variable part. So since W = W^1 , W is a first degree term. Since $7x^2$ has a square on the variable, this is a second degree term. Notice the number does not influence the degree of a term. A constant like -4 has no variable so it is considered degree zero. Products of letters are tricky. We add the degree of all the letters. So since $3a^2bc = 3a^2b^1c^1$, the degree is 2+1+1 = 4. It is a 4th degree term.

<u>Try the following examples with your instructor. For each term, identify the numerical coefficient and the variable part (if it has one). Also give the degree of the term</u>.

Example 1: $-8z^3$

Example 2: r^2

Example 3: -15

Example 4: $11\,b$

One of the key things to know about terms is that we can only add or subtract terms with the same variable part. So we can only add or subtract x with x and r^2 with r^2 and so on. Terms with the same variable part are called "like terms." To add or subtract like terms we add or subtract the numerical coefficients and keep the variables (letters) the same.

Look at the example of 5a+3a. Are these like terms? They both have the exact same letter part, so they are like terms. Think of it like 5 apples plus 3 apples. We would have 8 apples, not 8 apples squared or 8 double apples. So 5a + 3a = 8a. We can combine the like terms and keep the letter part the same.

Look at the example of 7a + 2b. Are these like terms? Since they have different letter parts, they are not like terms. Hence we cannot add them. Think of it like 7 apples plus 2 bananas. That will not equal 9 apple/bananas. It is just 7 apples and 2 bananas. That is a good way of thinking about adding or subtracting terms that don't have the same letter part. Hence 7a + 2b = 7a + 2b. They stay separate. In fact, many formulas have two or more terms that cannot be combined. 7a + 2b is as simplified as we can make it.

We even have special names for formulas that tell us how many terms it has. A formula with only 1 term is called a "monomial". A formula with exactly 2 terms is called a "binomial". A formula with exactly 3 terms is called a "trinomial".

<u>Try and simplify the following formulas with your instructor. After it is simplified, count how many terms the simplified form has. Then name the formula as a monomial, binomial or trinomial</u>.

Example 5: $5w-8w$

Example 6: $4m+9$

Example 7: $^{-}3p+{}^{-}9q+5p-4q$

Example 8: $6x^2+8x-14$

Practice Problems Section 5A

For each term, identify the numerical coefficient and the variable part (if it has one). Also give the degree of the term.

1. $9L$
2: $-3r^4$
3: 18

4. y^3
5: $-r^2$
6: $-12p$

7. $23v^5$
8: x^7
9: 5^2

10. $-w^2$
11: $-19h^2$
12: $3abc$

13. $-3x^2y^2$
14: m^2n
15: $-vw^2$

16. $-7b^3$
17: y^6
18: -17

19. $-p^8$
20: $19k^5$
21: $-13wxy^2$

22. $19a^3b^2$
23: p^4q^2
24: $-v^2w^2$

Simplify the following formulas by adding or subtracting the like terms if possible. Count how many terms the simplified form has and then name the formula as a monomial, binomial or trinomial.

25. $^{-}3a+11a$
26. $12m+7m$
27. $^{-}6v-{}^{+}14v$

28. $5x-14x$
29. $17m+9m-8m$
30. $^{-}13p+{}^{-}9p+5p$

31. $3y-8$
32. $6a+4b+9b$
33. $^{-}2p+{}^{-}8p-3m$

34. $8v-17v-12v$
35. $4a+6b-8c$
36. $3g-7h+10g-5h$

37. $5w-8x+3y$
38. x^2+9x^2-7x+1
39. $-2w^2+4w-8w$

40. x^3+7x-9
41. $4m^3+9m^2-10m^3-7m^2$
42. $5y-7y^2+8y-3$

Section 5B – Multiplying Terms with Associative and Distributive Properties

Remember a term is a product of numbers and/or letters. A term can be a letter by itself or a number by itself or a product of numbers and letters. The key thing to remember about terms is that we can only add or subtract "like" terms, that is ones that have the exact same letter part.

Many formulas have terms that cannot be added together. For example the perimeter of a rectangle is L+W+L+W which when simplified gives 2L+2W. Suppose we want to triple the size of a rectangle. This would triple the perimeter 3(2L+2W). How would we simplify this formula? This is what we will try to figure out in this section.

Let's start by thinking about multiplying terms. For example, look at 4(5y). The associative property says that a(bc)=(ab)c. This means that $4(5y)=(4\times5)y=20y$. Notice we cannot add or subtract unlike terms, but we can always multiply terms since they are a product. The key is just to multiply the number parts together. Look at $(7a)(3b)$. This is the same as $7\times3\times a\times b$ or $21ab$. Notice also that when we multiply two terms, our answer is just one term.

<u>Multiply the following terms with your instructor</u>.

Example 1: $^{-}6(7x)$

Example 2: $9a(^{-}4b)$

Example 3: $(^{-}8m)(^{-}6n)(2p)$

Let's look at the example of tripling the perimeter of a rectangle: 3(2L + 2W). The problem again is that 2L and 2W are not like terms so they cannot be added. So how do we multiply by 3? The answer is by using the distributive property. The distributive property is a(b+c) = ab+ac. So to multiply 3(2L + 2W) , we multiply 3 times 2L and 3 times 2W. Hence $3(2L+2W)=3\times2L+3\times2W=6L+6W$. So the formula for triple the perimeter is 6L+6W.

Let's try another example. $-4(2a-7b)$. It is often helpful to rewrite subtraction as adding a negative. In this example $-4(2a-7b)=-4(2a+{}^{-}7b)$. Now we use the distributive property to multiply. $-4(2a-7b)=-4(2a+{}^{-}7b)={}^{-}4\times 2a+{}^{-}4\times{}^{-}7b={}^{-}8a+{}^{+}28b$. So the simplified formula is ${}^{-}8a+28b$.

Sometimes we want to subtract a parenthesis. For example, look at $-({}^{-}3x+8)$. The key is to subtract all the terms inside the parenthesis. This is often called distributing the negative. You can also think of it like multiplying by -1. When we distribute the negative we get the following:

$-({}^{-}3x+8)=-{}^{-}3x-{}^{+}8={}^{+}3x-{}^{+}8=3x-8$. Notice that when we distribute the negative all the signs of the terms inside the parenthesis have changed to the opposite. -3x became +3x and +8 became -8.

Let's look at a more complicated example of simplifying. $-6(9y-4)-5(-2y+8)$. In problems like this, order of operations comes into play. In order of operations, we simplify parenthesis first. 9y-4 and -2y+8 are both as simplified as possible. They are not like terms so we cannot add or subtract them. Next we do the multiplications. It is very helpful to rewrite subtractions as adding the negative. So $-6(9y-4)-5(-2y+8)={}^{-}6(9y+{}^{-}4)+{}^{-}5({}^{-}2y+8)$. Now we use the distributive property to multiply.

$${}^{-}6(9y+{}^{-}4)+{}^{-}5({}^{-}2y+8)={}^{-}6\times 9y+{}^{-}6\times{}^{-}4+{}^{-}5\times{}^{-}2y+{}^{-}5\times{}^{+}8={}^{-}54y+{}^{+}24+{}^{+}10y+{}^{-}40$$

We are not quite finished since now there are some like terms we can add. Remember the y terms with the y terms and the constants with the constants.

$${}^{-}54y+{}^{+}24+{}^{+}10y+{}^{-}40={}^{-}44y+{}^{-}16$$

So when simplified completely this formula is -44y-16. Remember adding -16 is the same as subtracting 16.

Simplify the following with your instructor by using the distributive property. Remember to simplify completely.

Example 4: $7(9y+6)$

Example 5: $^{-}8(3m-5n)$

Example 6: $^{-}2(x+3w-7y)$

Example 7: $5(x+9)-(^{-}8x-11)$

Practice Problems Section 5B

Simplify the following using the associative property. Be sure to simplify completely.

1. $3(8k)$
2. $^{-}2(^{-}17y)$
3. $5(^{-}13bc)$
4. $a(^{-}4x^2)$
5. $^{-}7a(12c)$
6. $^{-}13wx(y)$
7. $^{-}22m(3n^2)$
8. $^{-}4x^2(5y^2)$
9. $^{-}10p^4(^{-}6q^4)$
10. $11k(^{-}7L)$

11. $^{-}15\left(x^3yz\right)$

12. $2r\left(-u^2v\right)$

13. $^{-}9n^3\left(2m^2\right)$

14. $-w\left(23x^2\right)$

15. $1.3\left(^{-}2.7u\right)$

16. $^{-}5.02\,\mathrm{v}\left(3.4y^3\right)$

17. $^{-}0.25y\left(^{-}8.4z^4\right)$

18. $\frac{1}{2}m\left(\frac{1}{3}np\right)$

19. $-\frac{2}{5}x\left(\frac{5}{7}w\right)$

20. $-\frac{4}{9}p\left(-\frac{27}{32}r\right)$

Simplify the following using the distributive property. Be sure to simplify completely.

21. $5(a+3)$

22. $^{-}2(y-9)$

23. $4\left(^{-}3b+4\right)$

24. $^{-}3\left(^{-}10x-6\right)$

25. $12(3v-8)$

26. $^{-}11(3x+5y)$

27. $6(2y+1)$

28. $^{-}4(2a-7b)$

29. $9(2x-4)$

30. $^{-}12\left(^{-}5d-9\right)$

31. $11(4w-12)$

32. $^{-}18(3v+5w)$

33. $7a(b+4)$

34. $^{-}2c\left(^{-}3d+13\right)$

35. $7(3x+y-6)$

36. $-(3p+18)$

37. $-\left(^{-}14a+5b-1\right)$

38. $-(8g-6h-4)$

39. $6(3y+12)-14y$

40. $^{-}2(5a-9b)+9a-4b$

41. $7(2x-10)+66$

42. $^{-}12\left(^{-}2d-3\right)-23d-34$

43. $13(3w-11)-(24w+20)$

44. $^{-}8(v+3w)-(5v-14)$

45. $7a(b+4)+2ab$

46. $^{-}2c\left(^{-}7d+13\right)+26c$

47. $7(3x+y)+2(x-y)$

48. $4p-(3p+18)$

49. $a+5b-\left(^{-}14a+5b-1\right)$

50. $^{-}7\left(^{-}4g+2h+1\right)-(8g-6h-4)$

Section 5C – Solving Equations with the Addition Property

Solving equations is a useful tool for determining quantities. In this section we are going to explore the process and properties involved in solving equations.

For example, in business we look at the break-even point. This is the number of items that need to be sold in order for the company's revenue to equal the cost. This is the number of items that must be sold so that the company is not losing money and is therefore starting to turn a profit. For example a company that makes blue-ray DVD players has costs equal to 40x+12000 where x is the number of blue-ray DVD players made. The equipment needed to make the DVD players was $12000 and it costs about $40 for the company to make 1 DVD player. They sell the DVD players for $70 so their revenue is 70x where x is the number of players sold. The break-even point will be where costs = revenue (40x + 12000 = 70x). How many DVD players do they need to sell to break even?

To solve problems like this we need to learn how to solve equations like 40x + 12000 = 70x. To solve an equation, we are looking for the number or numbers we can plug in for the variable that will make the equation true. For example, try plugging in some numbers for x in the break-even equation and see if it is true. If we plug in 100, we get the following: $40(100)+12000=70(100)$. But that is not true! $4000+12000 \neq 7000$ so 100 is not the solution. If we plug in 400 we get the following: $40(400)+12000=70(400)$. This is true since $16000+12000=28000$. The two sides are equal! So the company needs to make and sell 400 blue-ray DVD players to break even. After 400, they will start to turn a profit.

As you can see sometimes we can guess the answer to an equation. If you cannot guess the answer, then we need to have ways of figuring out the answer.

Addition Property of Equality

How do we find the answer to an equation when we cannot guess the answer? One property that is very helpful is the addition property. The addition property says that we can add or subtract the same number or term to both sides of an equation and the equation will remain true.

For example look at the equation $w+19={}^{-}4$. You may or may not be able to guess the number we can plug in for w that makes the equation true. The key is to add or subtract something from both sides so that we isolate the variable w. In equation solving, it is all about opposites. Do the opposite of what is being done to your letter. Since we are adding 19 to our variable w, let's subtract 19. Look what happens if we subtract 19 from both sides and simplify.

$w + 19 - 19 = {}^{-}4 - 19$

$w + 0 = {}^{-}4 + {}^{-}19$

$w = {}^{-}23$

First notice, that we had to subtract the same number from both sides. If you only subtracted 19 from the left side of the equation, the equation would no longer be true. Also subtracting 19 is the same as adding -19 from both sides. This helps when dealing with negative numbers. This shows us that the number we can plug in for w that makes the equation true is -23. How can we check if that is the correct answer? We plug in -23 for w in the original equation and see if the two sides are equal. ${}^{-}23 + 19 = {}^{-}4$ is true so -23 is the correct answer! Notice also that this is a conditional equation and is only true when w = -23 and false for any other number.

<u>Try to solve the following equations with your instructor using the addition property. Be sure to check your answers</u>.

Example 1: $w - 8 = {}^{-}15$

Example 2: $y - \frac{1}{6} = \frac{2}{3}$

Example 3: $0.345 = p + 1.56$

Practice Problems Section 5C

Solve the following equations with the addition property.

1. $x+9=23$
2. $m-12={}^{-}5$
3. $y-6={}^{-}17$
4. $13=v+{}^{-}7$
5. ${}^{-}18=w-15$
6. $h-13={}^{-}24$
7. $m+0.13=0.58$
8. $n-\frac{1}{2}=\frac{3}{8}$
9. ${}^{-}1.19={}^{-}2.41+T$
10. $\frac{4}{5}=\frac{2}{7}+c$
11. $x-74={}^{-}135$
12. $y+{}^{-}53={}^{-}74$
13. $c-8.14=6.135$
14. $d+\frac{1}{5}=\frac{3}{4}$
15. ${}^{-}630={}^{-}440+p$
16. $39=y+86$
17. $n+0.0351={}^{-}0.0427$
18. $-\frac{1}{10}=\frac{3}{4}+L$
19. $a+4.3=-1.2$
20. $\frac{4}{5}+m=\frac{1}{2}$
21. $\frac{5}{6}+P=\frac{1}{3}$
22. $5.07=d-8.1$
23. $267=x-243$
24. $y+14{,}320=145{,}208$
25. $y+7=-29$
26. $-67=p-78$
27. $w-\frac{7}{8}=\frac{5}{12}$
28. $t-\frac{1}{9}=-\frac{3}{4}$
29. $x-\frac{4}{3}=\frac{2}{7}$
30. $5.013+y=-2.13$
31. $2.3=\frac{23}{10}+y$
32. $7221+u=18{,}239$
33. $q-45.2=-27$
34. $\frac{9}{8}=\frac{1}{3}+w$
35. $0.4+x=\frac{1}{2}$
36. $f-\frac{3}{7}=\frac{9}{4}$
37. $m+\frac{8}{7}=-\frac{2}{9}$
38. $p+12.8=-4.97$
39. $\frac{3}{4}+x=0.75$
40. $8.001=m-0.12$

Section 5D – Solving Equations with the Multiplication Property of Equality

Look at the equation 4c = 17. You probably cannot guess what number we can plug in for the variable that will make the equation true. Also subtracting 4 will not help. If we subtract 4 we will get 4c – 4 = 17-4. That equation is more complicated, not less. This equation requires the multiplication property in order to solve it.

The multiplication property of equality says that we can multiply or divide both sides of the equation by any non-zero number. Remember, the key to equation solving is doing the opposite of what is being done to your variable. In 4c =17 , the variable is being multiplied by 4, so we should do the opposite. The opposite of multiplying by 4 is dividing by 4. Dividing both sides by 4 gives us the following.

$$4c = 17$$
$$\frac{4c}{4} = \frac{17}{4}$$

4 divided by 4 is 1 and 17 divided by 4 is 4.25 so we get the following:

$$4c = 17$$
$$\frac{{}^{1}\cancel{4}c}{\cancel{4}} = \frac{17}{4}$$
$$c = 4.25$$

Hence the answer is 4.25. Again we can check it by plugging in 4.25 into the original equation and seeing if it is equal. 4(4.25) = 17

Let's look at another example. $\frac{y}{7} = 2.5$

Since we are dividing our variable by 7, we should multiply both sides by 7 in order to isolate the variable.

$$7\left(\frac{y}{7}\right) = 7(2.5)$$
$${}^{1}\cancel{7}\left(\frac{y}{\cancel{7}}\right) = 7(2.5)$$
$$1y = 17.5$$

Since 7/7 = 1 and 7x2.5 = 17.5 and 1y = y, we are left with y = 17.5

Let's look at a third example. $-\frac{2}{3}w=\frac{7}{8}$

The variable is being multiplied by -2/3 so we need to divide both sides by -2/3. If you remember from fractions, dividing by a fraction is the same as multiplying by the reciprocal. So dividing by -2/3 is the same as multiplying by -3/2. So we are going to multiply both sides by -3/2.

$$-\frac{3}{2}\left(-\frac{2}{3}w\right)=-\frac{3}{2}\left(\frac{7}{8}\right)$$

Notice the reciprocals multiply to positive 1. So we are left with an answer of $-\frac{21}{16}$ or $-1\frac{5}{16}$.

$$-\frac{3}{2}\left(-\frac{2}{3}w\right)=-\frac{3}{2}\left(\frac{7}{8}\right)$$
$$+\frac{6}{6}w=-\frac{21}{16}$$
$$1w=-\frac{21}{16}$$
$$w=-1\frac{5}{16}$$

<u>Solve the following equations with your instructor by using the multiplication property</u>.

Example 1: $-8b=168$

Example 2: $\frac{x}{17}=-3$

Example 3: $-\frac{3}{5}w=\frac{1}{4}$

Practice Problems Section 5D

Solve the following equations. Simplify all fractions completely.

1. $2x = 22$
2. $28 = {}^{-}4y$
3. $12m = {}^{-}72$
4. ${}^{-}8 = 40n$
5. ${}^{-}9w = -144$
6. ${}^{-}65 = {}^{-}5y$
7. $6m = 90$
8. ${}^{-}98 = {}^{-}7a$
9. $14v = -70$
10. $13 = {}^{-}39n$
11. $-120d = -20$
12. $180 = {}^{-}15x$
13. $34 = {}^{-}2h$
14. $12g = 240$
15. ${}^{-}51 = {}^{-}3L$
16. $14f = -70$
17. $5 = 20x$
18. $36c = {}^{-}4$
19. $\frac{2}{7}y = \frac{3}{5}$
20. $\frac{4}{5} = -\frac{12}{25}u$
21. $-\frac{1}{9}u = -\frac{5}{18}$
22. $\frac{4}{9} = -7y$
23. $3b = -\frac{1}{5}$
24. $-\frac{4}{11} = \frac{16}{33}L$
25. $-\frac{3}{8}T = \frac{9}{8}$
26. $-\frac{3}{14} = -\frac{3}{7}h$
27. $\frac{6}{13}p = -\frac{14}{13}$
28. $0.4m = 5.2$
29. $0.3 = {}^{-}0.24a$
30. $1.2w = {}^{-}0.144$
31. $1.8 = {}^{-}0.09p$
32. ${}^{-}0.1d = 0.037$
33. ${}^{-}0.47 = {}^{-}2.35x$
34. $6.156 = {}^{-}1.8n$
35. $0.004c = {}^{-}0.2$
36. $0.2312 = 6.8b$
37. $0.035g = {}^{-}0.056$
38. ${}^{-}12 = 0.05f$
39. ${}^{-}0.33m = {}^{-}8.58$
40. $35 = -5n$
41. $15h = 75$
42. $\frac{7}{8} = \frac{2}{3}x$
43. $-\frac{1}{5}p = -\frac{7}{15}$
44. $\frac{8}{15} = \frac{7}{12}y$
45. $2.4m = -0.288$
46. $\frac{b}{5} = -45$
47. $\frac{n}{7} = 14$
48. $0.0005 = 0.0003x$
49. $-\frac{m}{12} = 4$
50. $9.2x = -64.4$

Section 5E – Steps to Solving General Linear Equations

A linear equation is an equation where the variable is to the first power. If a variable has an exponent of 2 (square) or 3 (cube) or higher, then it will require more advanced methods to solve the problem. In this chapter we are focusing on solving linear equations, but first we need to talk about the different types of equations.

<u>3 Types of Equations</u>

There are 3 types of equations: conditional, contradiction and identity.

Conditional equations are only true sometimes. Look at the equation $x + 4 = 11$. This type of equation is a conditional equation because it is only true if x = 7 and is not true if x is any other number. When an equation has a finite number of solutions, it is conditional. Most equations are conditional. Our job is to find the number we can plug in to make the equation true.

Contradiction equations are never true. Look at the equation w+3 = w+5 . This equation is never true. No matter what number we replace w with, we always get a false statement. When an equation has no solution it is called a contradiction equation. When asked to solve a contradiction, something weird will happen. When we subtract w from both sides we are left with 3 = 5 . This is not true. So you know the equation has "no solution". Look at another example $3b + 7 = 3b - 2$. A technique in solving equations is to bring the variable terms to one side. But if we subtract 3b from both sides we get +7 = -2. That is <u>never</u> true!! This tells us that the equation is never true no matter what. Hence this is a contradiction equation and the answer is "No Solution".

The third type of equation is an identity equation. This is one that is always true. Look at y+4 = y+4. We can plug in any number we want for y and it will be true. For example we could plug in 70 and see that 70+4=70+4 (true). We could plug in -549 and see that -549+4 = -549+4. Every time we plug in any number we get a true statement. The solution to an identity equation is "all real numbers", since it has infinitely many solutions. When solving an identity equation like y+4 = y+4 , we will subtract y from both sides, but then all the variables are gone and we are left with the true statement 4=4. When that happens you know the answer is "all real numbers". Look at another example $5a + 1 = 5a + 1$. Did you notice the two sides are exactly the same? If not, we can try to bring the variable terms to one side. But if we subtract 5a from both sides we get +1 = +1. That is <u>always</u> true!! This tells us that the equation is true no matter what we plug in for a. Hence this is an identity equation and the answer is "All Real Numbers".

<u>General Equation Solving</u>

We have seen that we can solve equations by guessing the answer. If we cannot guess we can use the multiplication and addition properties to help us figure out the answer. Let's now look at some more complicated equations and the steps to solving them.

Steps to Solving a Linear Equation *(It is Vital to Memorize These!!)*

1. Eliminate parenthesis by using the distributive property.
2. Eliminate fractions by multiplying both sides of the equation by the LCD.
3. Eliminate decimals by multiplying both sides of the equation by a power of 10 (10, 100, 1000…)
4. Use the addition property to eliminate variable terms so that there are only variables on one side of the equation.
5. Use the addition property to eliminate constants so that there are only constants on one side of the equation. The constants should be on the opposite side of the variables.
6. Use the multiplication property to multiply or divide both sides of the equation in order to isolate the variable by creating a coefficient of 1 for the variable.
7. Check your answer by plugging it into the original equation and see if the two sides are equal.

Note: After each step, always add or subtract like terms that lie on the <u>same</u> side of the equation.

Note: Remember that an equation can have "no solution" or "all real numbers" as a solution in the cases of contradiction and identity equations.

Let's look at another example equation $5x+7=4x-3$. When dealing with an equation like this, our goal is to bring letters to one side and the constant numbers to the other side. If we want to eliminate the 4x on the right side, we can subtract 4x from both sides.

$$5x+7=4x-3$$
$$-4x \qquad -4x$$
$$x+7=0-3$$
$$x+7={}^{-}3$$

Notice that there are only x variables on the right side. Can you guess the answer now? If not we can get rid of the 7 by subtracting 7 (adding -7) to both sides.

$$x+7={}^{-}3$$
$${}^{-}7 \qquad {}^{-}7$$
$$x+0={}^{-}10$$
$$x={}^{-}10$$

Notice the number we can plug in for x that makes the equation true is -10. Check if that is the correct answer. Plugging in -10 into the original equation we get the following:

$$5\left({}^{-}10\right)+7=4\left({}^{-}10\right)-3$$
$$-50+7=-40-3$$
$$-43=-43$$

So when we plug in -10, we do get a true statement. Hence -10 is the solution. Note that the two sides were equal and both equal to -43, but -43 is <u>not</u> the solution. The solution is the number we replaced the letter with that made the two sides equal. Also notice this was a conditional equation. It was only true when x = -10 and false otherwise.

Let's look at an example $-6\left(2w-8\right)+4w=3w-(7w+9)$

Step 1: Our first step is to distribute and eliminate parenthesis, so we will multiply the -6 times both the 2w and the -8. We will also distribute the negative to the 7w and the 9 and eliminate that parenthesis as well. We should only distribute to the terms in the parenthesis. For example we do <u>not</u> distribute the -6 to the 4w since the 4w is not in the parenthesis.

$$-6\left(2w-8\right)+4w=3w-(7w+9)$$
$$-12w+48+4w=3w-7w-9$$

Always look to simplify after each step. For example in this problem the -12w and 4w are like terms on the same side. Also the 3w and -7w are also like terms on the same side. If you struggle with negatives, you can convert the -7w to adding the opposite. Be careful. <u>Do not add or subtract terms on opposite sides of the equation</u>.

$$-6\left(2w-8\right)+4w=3w-(7w+9)$$
$$-12w+48+4w=3w-7w-9$$
$$-8w+48=-4w-9$$

Step 2 and 3: There are no fractions or decimals so we proceed directly to step 4.

Step 4: We need to bring the variables to one side. You can bring variables to either side, but many students like to bring the variables to the left side only. So we will need to eliminate the -4w on the right side. Hence we will add the opposite +4w to both sides.

$$-8w+48=-4w-9$$
$$+4w \qquad\qquad +4w$$
$$-4w+48=0-9$$
$$-4w+48=-9$$

Step 5: We need to bring the constants to the opposite side as the variables. So we need to get rid of the +48. Hence we will subtract 48 (add -48) to both sides. We are then left with -4w= -57

$$\begin{aligned} -4w+48 &= -9 \\ -48 \quad & \quad -48 \\ -4w+0 \quad &= -57 \\ -4w &= -57 \end{aligned}$$

Step 6: We now need to get the *w* by itself. Since the *w* is being multiplied by -4, we will divide both sides by -4 to get our answer of 57/4. Notice the answer can be written three ways and all are equally correct $\left(\frac{57}{4} or 14\frac{1}{4} or 14.25\right)$

$$\begin{aligned} -4w &= -57 \\ \frac{\cancel{-4}^{1}}{\cancel{-4}}w &= \frac{-57}{-4} \\ 1w &= 14.25 \\ w &= 14.25 \end{aligned}$$

Step 7: Let's check our answer by plugging into the original equation. Notice that all of the w's have to be replaced with 14.25 and don't forget to use the order of operations when simplifying each side. As you can see, checking your answer can be just as much work as solving the equation.

$$\begin{aligned} -6(2w-8)+4w &= 3w-(7w+9) \\ -6(2\times14.25-8)+4\times14.25 &= 3\times14.25-(7\times14.25+9) \\ -6(28.5-8)+4\times14.25 &= 3\times14.25-(99.75+9) \\ -6(20.5)+4\times14.25 &= 3\times14.25-(108.75) \\ -123+57 &= 42.75-(108.75) \\ -66 &= -66 \end{aligned}$$

Let's try another example. Look at $\frac{1}{3}c-\frac{3}{5}=\frac{1}{2}c+4$

Step 1: There are no parenthesis so we proceed to step 2.

Step 2: To eliminate fractions we find the LCD. Since the denominators are 3,5 and 2 the LCD is 30. Hence we will multiply everything on both sides by 30. This will eliminate the fractions. Remember all the terms must be multiplied by 30. When multiplying a whole number (30) by fractions it is good to write the whole number as a fraction (30/1).

$$\frac{30}{1}\left(\frac{1}{3}c-\frac{3}{5}\right)=\frac{30}{1}\left(\frac{1}{2}c+4\right)$$
$$\frac{30}{1}\times\frac{1}{3}c-\frac{30}{1}\times\frac{3}{5}=\frac{30}{1}\times\frac{1}{2}c+\frac{30}{1}\times 4$$
$$\frac{30}{3}c-\frac{90}{5}=\frac{30}{2}c+\frac{120}{1}$$
$$10c-18=15c+120$$

Notice we are now left with an equation without fractions.

Step 3: There are no decimals, so we proceed to step 4.

Step 4: We bring all the variables to the left side by subtracting 15*c* (adding -15*c*) to both sides.

$$\begin{aligned}10c-18&=15c+120\\ -15c\qquad &\quad -15c\\ -5c-18&=0+120\\ -5c-18&=120\end{aligned}$$

Step 5: We bring all the constants to the opposite side. We can eliminate the -18 by adding +18 to both sides.

$$\begin{aligned}-5c-18&=120\\ +18\quad &\ \ +18\\ -5c+0&=138\\ -5c&=138\end{aligned}$$

Step 6: Isolate the variable. Since we are multiplying the w by -5, we divide both sides by -5.

$$-5c=138$$
$$\frac{\overset{1}{\cancel{-5}}}{\cancel{-5}}c=\frac{138}{-5}$$
$$1c=-\frac{138}{5}$$
$$c=-27\frac{3}{5}\text{ or }-27.6$$

Step 7: Check your answer. By plugging -27.6 into the original equation, the two sides are equal.

Let's try a third example: $0.24(3m-1)=0.82m+0.24-0.1m$

Step 1: We will distribute the 0.24 to the 3*m* and -1 to eliminate the parenthesis. We will make sure to combine any like terms that are on the same side. Notice 0.82*m* and -0.1*m* are like terms on the same side, so we can add them.

$$0.24(3m-1)=0.82x-0.24-0.1m$$
$$0.24\times 3m+0.24\times {}^{-}1=0.82m-0.24-0.1m$$
$$0.72m-0.24=0.72m-0.24$$

Step 2: There are no fractions, so we proceed to step 3.

Step 3: Since the most decimal places to the right is two (hundredths place), we will multiply everything on both sides by 100. If one of the decimals had ended in the thousandths place, we would multiply by 1000 and so on. Notice all the decimals are gone.

$$0.72m-0.24=0.72m-0.24$$
$$100(0.72m-0.24)=100(0.72m-0.24)$$
$$100\times 0.72m+100\times {}^{-}0.24=100\times 0.72m+100\times {}^{-}0.24$$
$$72m-24=72m-24$$

Step 4: Bring the variables to one side by subtracting 72*m* from both sides. Notice all variables cancel and we are left with -24 = -24 which is a true statement.

$$72m-24=72m-24$$
$$-72m \qquad -72m$$
$$0-24=0-24$$
$$-24=-24$$

Since we are left with a true statement and all the variables are gone, we need go no further. This is an always true equation (identity). So the answer is "All Real Numbers".

<u>Try to solve the following equations with your instructor</u>.

Example 1: $6d+8=5d-3$

Example 2: $3y+4=3y$

Example 4: $2d+7=5d-3d+7$

Example 5: $4(3d+7)=19(d-2)-5d+2$

Example 6: $\frac{1}{4}L-\frac{1}{2}=\frac{1}{3}L+\frac{5}{6}$

Example 7: $0.25(y+0.4)+0.2=0.15y-0.5$

(Remember to eliminate parenthesis <u>before</u> eliminating decimals)

<u>Practice Problems Section 5E</u>

Solve the following equations. If your answer is a fraction, be sure to simplify it completely. For #39, #40 and #47, remember to eliminate parenthesis before eliminating fractions or decimals.

1. $7y-15=20$
2. $^{-}18=4p+2$
3. $12=-1a-17$
4. $9+{}^{-}7n=8$
5. $-12x+7=-9x-11$
6. $5y-14=-3y+18$
7. $-1a+27=-6a-13$
8. $-2b+13=9b+7$

9. $6-13v={}^{-}5+{}^{-}11v$

10. ${}^{-}23+5m={}^{-}19-18m$

11. $13-y=-1y+17-4$

12. $14g-3={}^{-}3g+8$

13. $6a=5a-7$

14. $8b-13=8b+2$

15. $17v=16v-9$

16. ${}^{-}8b={}^{-}9b+4$

17. ${}^{-}8x+{}^{+}8x=13-13$

18. ${}^{-}5w={}^{-}6w+{}^{-}11$

19. ${}^{-}9a-13={}^{-}10a+4$

20. $1.3x-2.7=0.3x-3.4$

21. $16w+9-15w=w+9$

22. ${}^{-}5.2x+7.3={}^{-}6.2x-3.6$

23. $6a-\frac{1}{4}=5a+\frac{3}{4}$

24. $0.8b-2.57={}^{-}0.2b+2.9$

25. $a-0.004={}^{-}0.5a+0.053+0.5a$

26. $\frac{8}{5}x-\frac{1}{3}=\frac{3}{5}x+\frac{1}{3}$

27. $26x+19-21x=5x-17$

28. ${}^{-}5.2d+7.3={}^{-}6.2d-3.6$

29. $-31m+17+28m=-3m-10+27$

30. $25+5n-24=-3n+18+8n$

31. $-2(4x+3)=-6x$

32. $7(y-3)+2y=6$

33. $-4(5\mathrm{a}-1)+18\mathrm{a}=-2(-3\mathrm{a}+2)$

34. $11b-(7b+9)=4(b-3)$

35. $\frac{1}{2}m+1=-\frac{1}{3}m-2$

36. $\frac{2}{3}n-\frac{1}{5}=\frac{2}{5}n$

37. $\frac{3}{4}c+2=1c-\frac{1}{2}$

38. $\frac{1}{6}d-\frac{3}{2}=\frac{1}{2}d+\frac{5}{6}$

39. $\frac{1}{5}\left(2p+\frac{1}{3}\right)=\frac{1}{2}\left(\frac{1}{3}p-1\right)$

40. $\frac{2}{3}(w+6)=\frac{1}{4}(w-8)$

41. $0.5x-0.3=0.8x+0.9$

42. $0.09p-0.04=0.09p+0.06$

43. $0.23x-0.4=0.38x+0.2$

44. $0.007y-0.03=0.009y+0.06$

45. $0.2m+0.41=0.18m+0.67$

46. $0.6n-2=n+0.3$

47. $-0.4(2a+1)=-0.9a+0.5+0.1a-0.9$

Section 5F – Solving Proportions

A special type of equation that has many applications are proportions. A "proportion" is when two fractions are equal to each other. For example $\frac{3}{5}=\frac{6}{10}$. Notice the two fractions are equal to each other. One way to check if two fractions are equal is by looking at the cross products. Notice the numerator of one fraction (3) times the denominator of the other fraction (10) is equal to the other cross product (6x5). Notice both cross products are equal to 30. This is an easy way to check if two fractions are equal.

In general if $\frac{c}{d}=\frac{e}{f}$ then $c\times f=e\times d$.

Try the following examples with your instructor. Determine if the two fractions are equal.

Example 1: $\frac{3}{7}=\frac{9}{22}$

Example 2: $\frac{8}{18}=\frac{12}{27}$

Often we need to solve a proportion though. Setting the cross products equal is an easy way to simplify a proportion equation and make it much easier to solve.

Let's look at an example. Solve the following proportion: $\frac{15}{16}=\frac{h}{8}$

First we notice that this is two fractions set equal, so it is a proportion. If it does not have this form, you need to use the methods discussed in section 5D. Since this is a proportion we can set the cross products equal to each other.

$$\frac{15}{16}=\frac{h}{8}$$
$$h\times 16=15\times 8$$

Simplifying gives us 16*h* = 120. Now we solve by isolating the variable (divide both sides by 16).

$$16h = 120$$
$$\frac{16h}{16} = \frac{120}{16}$$
$$\frac{\overset{1}{\cancel{16}}h}{\cancel{16}} = \frac{120}{16}$$
$$1h = 7.5$$
$$h = 7.5$$

At the beginning of this chapter, we talked about the famous z-score formula in statistics. $z = \dfrac{x-\mu}{\sigma}$. Can we find the x-value (pounds) for a z-score of -2.4? If $\mu = 9.5$ and $\sigma = 1.5$ can we back solve the formula and figure out what x needs to be?

Plugging in the correct values into the formula gives us the following:

$$z = \frac{x-\mu}{\sigma}$$
$$-2.4 = \frac{x-9.5}{1.5}$$

Now if we were to write -2.4 as -2.4/1 we would have a proportion! We could then cross multiply and solve. Let's try it.

$$z = \frac{x-\mu}{\sigma}$$
$$-2.4 = \frac{x-9.5}{1.5}$$
$$\frac{-2.4}{1} = \frac{x-9.5}{1.5}$$

Setting the cross products equal and solving gives us the x value.

$$\frac{-2.4}{1} = \frac{x-9.5}{1.5}$$
$$1(x-9.5) = -2.4(1.5)$$
$$x - 9.5 = -3.6$$
$$+9.5 \quad +9.5$$
$$x + 0 = 5.9$$
$$x = 5.9$$

So the x value for a z-score of -2.4 is 5.9 pounds.

Solve the following proportion problems with your instructor:

Example 1: $\frac{3.5}{b}=\frac{2}{5}$

Example 2: $\frac{y+6}{7}=\frac{y-4}{3}$

Practice Problems Section 5F

Solve the following proportion problems. Simplify all fraction answers completely.

1. $\frac{1}{x}=\frac{3}{5}$
2. $\frac{w}{-4}=\frac{1}{6}$
3. $\frac{-3}{7}=\frac{y}{4}$
4. $\frac{8}{5}=\frac{3}{2d}$
5. $\frac{7.5}{x}=\frac{2.5}{9}$
6. $\frac{3w}{7}=\frac{2}{3}$
7. $\frac{-4}{13}=\frac{-1}{m}$
8. $\frac{-11}{x}=\frac{22}{5}$
9. $\frac{8}{2.1}=\frac{w}{6.3}$
10. $\frac{1}{5w}=\frac{^{-}8}{15}$
11. $\frac{0.25}{4}=\frac{0.15}{L}$
12. $\frac{^{-}18}{25}=\frac{9}{^{-}5u}$
13. $\frac{0.4}{1.2}=\frac{^{-}0.8d}{2.4}$
14. $\frac{^{-}50}{3h}=\frac{20}{^{-}9}$
15. $\frac{3}{2.5}=\frac{1.8f}{7.5}$
16. $\frac{2p-1}{5}=\frac{3p}{10}$
17. $\frac{7}{x+4}=\frac{2}{x+9}$
18. $\frac{3w}{7}=\frac{2w+5}{4}$
19. $\frac{-5}{m-1}=\frac{-2}{m+3}$
20. $\frac{x+4}{6}=\frac{2x}{11}$
21. $\frac{w-7}{1.5}=\frac{w}{1.5}$
22. $\frac{6}{x+5}=\frac{4}{x-7}$
23. $\frac{w-14}{9}=\frac{3w}{17}$
24. $\frac{3}{1.5+k}=\frac{7}{4.5}$

Section 5G – Reading, Understanding, Solving and Graphing Inequalities

A topic that is very important in statistics, algebra, and in many sciences is being able to use, read and understand inequalities. A linear inequality looks a lot like an equation, but uses the inequality symbols $<$, $\leq$, $>$, $\geq$, or $\neq$. It is vital to memorize these symbols and know what they mean. For example, statistics students routinely get P-value problems wrong not because they don't understand P-value but because they don't know the meaning of " < ". Let's start by going over the meaning of each of these symbols and give examples using the symbols correctly.

" < " means "less than". Notice the symbol looks like an arrow pointing to the left. On the number line, smaller numbers are on the left as in $-2 < +10$. Notice the closed end of the inequality symbol points toward the smaller number. This is true for all of the inequality symbols. <u>The symbol points toward the smaller number</u>!

" $\leq$ " means "less than or equal to". Notice the symbol looks like the less than symbol " < " but now has an extra line below. "Less than or equal to" works like a "less than" symbol with the symbol pointing toward the smaller number. However "less than or equal to" now meets the added criteria that if the two numbers were equal it would still be true. For example: ${}^{-}2 \leq {}^{+}10$ is true and ${}^{+}9 \leq {}^{+}9$ is also true! (Notice that without the equal to part, ${}^{+}9 < {}^{+}9$ is <u>not</u> true.)

" > " means "greater than". Notice the symbol looks like an arrow pointing to the right, and on the number line, larger numbers are on the right. Look at the example ${}^{+}23 > {}^{+}7$. This is true. Notice the closed end of the inequality symbol points toward the smaller number $\left({}^{+}7\right)$ and the open end is toward the larger number $\left({}^{+}23\right)$. This is true for all of the inequality symbols. <u>The symbol points toward the smaller number</u> and the <u>open end opens toward the bigger number</u>.

" $\geq$ " means "greater than or equal to". Notice the symbol looks like the "greater than"

than symbol " > " but now has an extra line below. "Greater than or equal to" works like a "greater than" symbol with the symbol pointing toward the right. However "greater than or equal to" now meets the added criteria that if the two numbers were equal it would still be true. For example: ${}^{+}17 \geq {}^{-}1$ is true and ${}^{-}3 \geq {}^{-}3$ is also true! (Notice that without the equal to part, ${}^{-}3 > {}^{-}3$ is <u>not</u> true.)

“ ≠ “ means “not equal to”. Notice the symbol looks like an equal sign with a line drawn through it. This symbol is only true if the two numbers are not equal. For example $^{+}6 \neq {}^{+}11$ is true but $^{-}9 \neq {}^{-}9$ is <u>not</u> true!

<u>Try the following inequality problems with your instructor</u>.

Directions: Identify which number is larger and which number is smaller or if the two numbers are equal. Then determine if the symbol is used <u>correctly</u> or <u>incorrectly</u>?

Example 1: $^{-}10 > {}^{+}2$

Example 2: $^{+}15 \geq {}^{+}4$

Example 3: $^{-}7 < {}^{+}22$

Example 4: $^{-}13 \leq {}^{-}13$

Example 5: $^{+}5.3 \neq {}^{+}7.6$

Inequalities are also frequently used in algebra. For example look at the inequality $"x > 3"$. What does this mean? It is not comparing two numbers like the last examples. There is a variable involved. The most important idea to remember is the following. When we solve an equation, we are trying to find out what number or numbers we can replace the letter by so that the statement will be true. An inequality works the same way.

Ask yourself the following question. What numbers can we replace the x with that makes the inequality "$x > 3$" true? Would 4 work? If we replace x with 4 we get 4 > 3. Is that true? Yes. Then 4 is one of the solutions to the inequality $x > 3$. But is 4 the only solution? Wouldn't 3.001 or 4.5 or 5 or 10.6 or 100 or 212.37 also work? They sure would. Any number greater than 3 would work. So "$x > 3$" represents "all numbers greater than 3".

Can we graph "all numbers greater than 3" on the number line? What would it look like?

We will draw a number line and shade all the numbers greater than 3. Be careful, it is not just whole numbers, it includes all fractions or decimals greater than 3 as well. In other words all real numbers greater than 3.

(=================>

$^{-}2 \quad ^{-}1 \quad 0 \quad ^{+}1 \quad ^{+}2 \quad ^{+}3 \quad ^{+}4 \quad ^{+}5 \quad ^{+}6 \quad ^{+}7$

Notice that the graph starts at 3 and shades all the number to the right. Notice also that it begins with a parenthesis. When graphing on the number line, a parenthesis means that the starting number $\left(^{+}3\right)$ is <u>not</u> included. Remember the number 3 is <u>not</u> part of the solution. If we plug in 3 into the inequality $x > 3$ we would get $3 > 3$ which is <u>not</u> true.

Let's try to graph another example. Let's graph $x \le 5$. What does this mean? The key is to ask yourself what numbers you can replace x with that will give you a true statement? Will 8 work? No. $8 \le 5$ Is not true. How about 1? Yes. 1 is a solution since $1 \le 5$ is a true inequality. Is 1 the only solution? Wouldn't 4.999 or 4 or 2.7 or 0 or $^{-}164.9$ also work? They sure would. Plugging any of those numbers in for x will give a true statement. So we see that " $x \le 5$" represents "all real numbers less than or equal to 5. So we want to shade all the numbers to left of 5 on the number line.

< ============================]

$^{-}2 \quad ^{-}1 \quad 0 \quad ^{+}1 \quad ^{+}2 \quad ^{+}3 \quad ^{+}4 \quad ^{+}5 \quad ^{+}6 \quad ^{+}7$

Notice that the graph starts at 5 and shades all the number to the left. Notice also that it begins with a bracket. When graphing on the number line, a bracket means that the starting number $\left(^{+}5\right)$ <u>is</u> included. The key is that the number 5 <u>is</u> part of the solution. If we plug in 5 into the inequality $x \le 5$ we would get $5 \le 5$ which <u>is</u> true. So " $x \le 5$" means "all real numbers less than or equal to 5".

Note: What happens if the variable is on the right hand side?

Inequalities really can be confusing when the variable is on the right hand side. For example look at $-1 \ge x$. What does this mean? The key is still the same. What numbers can we replace x with that make this inequality true? Will ${}^{+}7$ work? If we plug in ${}^{+}7$ into the inequality $-1 \ge x$ we get ${}^{-}1 \ge {}^{+}7$. This is <u>not</u> true! ${}^{-}1$ is less than ${}^{+}7$. Let's try another. Will ${}^{-}4$ work? If we plug in ${}^{-}4$ into the inequality $-1 \ge x$ we get ${}^{-}1 \ge {}^{-}4$. This <u>is</u> true! ${}^{-}1$ is greater than ${}^{-}4$. (A person that loses only $1 is richer than a person that loses $4.)
What about ${}^{-}1.2$ or ${}^{-}3$ or ${}^{-}12.5$ or ${}^{-}125$ or ${}^{-}507.3$? Won't they also make the inequality $-1 \ge x$ true? Definitely. So what have we learned? Even though the inequality $-1 \ge x$ uses a "greater than or equal to" symbol, in reality, "$-1 \ge x$" means "all real numbers <u>less</u> than or equal to ${}^{-}1$".

< ====================]

$^{-}6 \quad {}^{-}5 \quad {}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1$

Sometimes we like to represent all real numbers in between two numbers. Look at the inequality ${}^{-}4 < x < {}^{+}2$. Notice the smallest number is on the left and the larger number is on the right. This is just like the number line. The key question again is what numbers can we replace x with that makes the inequality true? Will ${}^{+}5$ work? No, ${}^{+}5$ is <u>not</u> in between ${}^{-}4$ and ${}^{+}2$ on the number line. How about 0? Yes 0 <u>is</u> in between ${}^{-}4$ and ${}^{+}2$ on the number line. Can you name some more numbers in between? How about ${}^{-}3.5$ or ${}^{-}2\frac{5}{8}$ or ${}^{+}1.997$? These would also make the inequality ${}^{-}4 < x < {}^{+}2$ true. So the inequality "${}^{-}4 < x < {}^{+}2$" represents all real numbers in between ${}^{-}4$ and ${}^{+}2$ on the number line.

(======================)

$^{-}6 \quad {}^{-}5 \quad {}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1 \quad {}^{+}2 \quad {}^{+}3 \quad {}^{+}4$

Notice again that it begins and ends with a parenthesis. When graphing on the number line, a parenthesis means that the starting number $\left({}^{-}4 \text{ and } {}^{+}2\right)$ are <u>not</u> included. If we plug in ${}^{-}4$ or ${}^{+}2$ into the inequality ${}^{-}4 < x < {}^{+}2$ we would get statements that are <u>not</u> true.

How would the graph be different if the inequality was written as ${}^{-}4 \le x < {}^{+}2$? Well notice that now ${}^{-}4$ <u>is</u> now a solution, but ${}^{+}2$ is still not. So we will need to use a bracket on ${}^{-}4$ and a parenthesis on ${}^{+}2$.

(======================]

$^{-}6 \quad {}^{-}5 \quad {}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1 \quad {}^{+}2 \quad {}^{+}3 \quad {}^{+}4$

What do you think would happen if the inequality was $^-4 \le x \le {}^+2$? If you said there would be brackets on both ends, you are absolutely right!

Sometimes you will see a not equal statement like $x \ne {}^+5$ in algebra classes, but what does this mean? Think about what you can replace x with that would make it true. Notice you can plug in any number you want for x except $^+5$. So $x \ne {}^+5$ means "all real numbers except $^+5$. What would the graph of $x \ne {}^+5$ look like? Shade everything on the number line, but leave a parenthesis at $^+5$.

<============================)(=======>

$^-2$ $^-1$ 0 $^+1$ $^+2$ $^+3$ $^+4$ $^+5$ $^+6$ $^+7$

<u>Try the following inequality problems with your instructor</u>.

Directions: Graph the following inequalities on the number line. Be careful to use either parenthesis and/or brackets correctly and shade the correct direction.

Example 6: $x > {}^+5$

Example 7: $x \le {}^-8$

Example 8: $^-3 \ge x$

Example 9: $^-12 \le x < {}^+5$

Example 10: $x \ne {}^-7$

(Instructor examples continued.)

Directions: Explain each of the following graphs in words and write an inequality with *"x"* that represents it.

Example 11:

< =============)

$^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 $^{+}1$

Example 12:

[==========================>

$^{-}2$ $^{-}1$ 0 $^{+}1$ $^{+}2$ $^{+}3$ $^{+}4$ $^{+}5$ $^{+}6$ $^{+}7$

Example 13:

[===================]

$^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 $^{+}1$ $^{+}2$ $^{+}3$ $^{+}4$

Example 14:

<=========)(================================>

$^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 $^{+}1$ $^{+}2$ $^{+}3$ $^{+}4$

Sometimes it is necessary in algebra to solve an inequality for the variable. We are beginning to understand what inequalities like $x > 7$ means, but what if it was $3x+1>7$? What does that mean? The key is to solve the inequality as if it was an equation. Imagine in your mind that $3x+1>7$ is just like $3x+1=7$. How would you solve that equation? Using the steps to solving equations, we would subtract 1 from both sides, and then divide by 3. This is exactly what you would do with the inequality as well.

$$
\begin{aligned}
3x+1 &> 7 \\
-1 \quad & \quad -1 \\
3x &> 6 \\
\frac{3x}{3} &> \frac{6}{3} \\
1x &> 2 \\
x &> 2
\end{aligned}
$$

So we see that when we solve $3x+1>7$ for x we get $x>2$ which is all real numbers greater than 2.

<u>A few important notes about solving inequalities</u>

Though in most respects solving inequalities is the same as solving equations, there is 1 important difference. Look at the following example.

$^{-}5 <^{+} 7$ This is a true statement.

$^{-}5 +^{+} 6 <^{+} 7 +^{+} 6$ This is still true. When we add a positive number to both sides, the inequality remains true.

$^{-}5 - 13 <^{+} 7 - 13$ This is still true. When we subtract a number from both sides, the inequality remains true.

$^{-}5 +^{-} 10 <^{+} 7 +^{-} 10$ This is still true. When we add a negative number to both sides, the inequality remains true.

So to summarize. We can add or subtract any number we want from both sides of the inequality and the inequality will remain true. Let's look at another example.

$^{-}15 <^{+} 10$ This is a true statement.

$^{-}15 \times^{+} 4 <^{+} 10 \times^{+} 4$ This is still true. When we multiply both sides by a positive number, the inequality remains true.

$\frac{^{-}15}{^{+}5} < \frac{^{+}10}{^{+}5}$ This is still true. When we divide both sides by a positive number, the inequality remains true.

$^{-}15 \times^{-} 2 <^{+} 10 \times^{-} 2$ This is NOT true! When we multiply both sides by $^{-}2$ we get $^{+}30 < ^{-}20$. This is FALSE! So if you multiply both sides by a negative number, the inequality is false. Notice we can fix it. Just switch the inequality from $<$ to $>$ and we get true statement $^{+}30 < ^{-}20$.

$\frac{^{-}15}{^{-}5} < \frac{^{+}10}{^{-}5}$ This is also NOT true! When we divide both sides by $^{-}5$ we get $^{+}3 < ^{-}2$. This is FALSE! So if you divide both sides by a negative number, the inequality is false. Notice again we can fix it. Just switch the inequality from $<$ to $>$ and we get true statement $^{+}3 < ^{-}2$.

Rules for solving inequalities

- When solving inequalities, follow the same steps that we use to solve equations.
- If you add or subtract any number from both sides, the inequality remains true.
- If you multiply or divide both sides of the inequality by a positive number, the inequality remains true.
- If you multiply or divide both sides of the inequality by a negative number, the inequality is now false and must be switched to the opposite sign.
  $<$ changes to $>$
  $\leq$ changes to $\geq$
  $>$ changes to $<$
  $\geq$ changes to $\leq$

Note: Solving inequalities for variables in-between two numbers

We have seen inequalities like $^{-}3 < w \leq {}^{+}11$ where the variable is in-between two numbers. We saw that this represents all real numbers between $^{-}3$ and $^{+}11$, including $^{+}11$ but not including $^{-}3$. What if there is an expression in the middle instead of a single variable? What does that mean? The key again is to solve the inequality and isolate the variable. Since there are three sides, we will be performing our solving steps to all three sides.

Remember, if you multiply or divide all three sides by a negative number, you must switch both signs. Look at the following example.

$$^{-}15 < {}^{-}2w+3 \leq {}^{+}17$$

$^{-}15 < {}^{-}2w+3 \leq {}^{+}17$ Subtract 3 from all three sides (or add $^{-}3$ to all three sides)

$\quad {}^{-}3 \qquad {}^{-}3 \qquad {}^{-}3$

$$^{-}18 < {}^{-}2w+0 \leq {}^{+}14$$

$$^{-}18 < {}^{-}2w \leq {}^{+}14$$

$\frac{^{-}18}{^{-}2} > \frac{^{-}2w}{^{-}2} \geq \frac{^{+}14}{^{-}2}$ Divide all three sides by $^{-}2$ (Don't forget to switch the signs!!)

$$^{+}9 > w \geq {}^{-}7$$

Solve the following inequalities with your instructor. Explain the answer in words.

Example 15: $w+17 \geq {}^{-}13$

Example 16: $^{-}6v \leq 78$

Example 17: $8y-4 < {}^{-}20$

Example 18: $-\frac{3}{4}h+\frac{2}{3} < -\frac{1}{2}$

Example 19: $^{-}7 \leq {}^{-}4p+1 \leq {}^{-}19$

Practice Problems Section 5G

Solve the following proportion problems.

Directions: Identify which number is larger and which number is smaller or if the two numbers are equal. Then determine if the inequality symbol is used correctly or incorrectly?

1. $^{+}17 > {}^{+}12$
2. $^{-}5 \geq {}^{+}3$
3. $^{-}7 < {}^{-}11$
4. $^{+}17 \leq {}^{+}17$
5. $+\frac{1}{4} \neq {}^{+}0.25$
6. $^{-}25 > 0$
7. $^{+}7\frac{5}{6} \geq {}^{+}7$
8. $^{-}84 < {}^{-}6$
9. $^{-}21 > {}^{-}21$

10 $^{+}0.274 \neq {}^{+}1.35$

11. $^{-}19 > {}^{+}14$
12. $^{+}52 \geq {}^{+}52$
13. $-\frac{3}{4} < +\frac{1}{7}$
14. $^{-}13 \leq {}^{-}13$
15. $^{+}5.3 \neq {}^{+}7.6$
16. $^{+}5.3 \neq {}^{+}7.6$
17. $-9 \leq 0 < {}^{+}14$
18. $^{+}3 \leq {}^{-}10 \leq {}^{+}21$
19. $^{-}23 < {}^{-}23 \leq 0$
20. $^{+}3.9 < {}^{+}5.2 \leq {}^{+}5.2$
21. $^{-}7 \neq {}^{+}3$

Directions: Graph the following inequalities on the number line. Your number line should be labeled. Be careful to use either parenthesis and/or brackets correctly and shade the correct direction.

22. $x > {}^{+}6$
23. $x \leq {}^{-}12$
24. $^{-}13 \leq x$
25. $x < {}^{+}8$
26. $x \geq {}^{-}11$
27. $^{+}9 \geq x$
28. $x < {}^{+}2.5$
29. $x \geq \frac{1}{4}$
30. $-3\frac{1}{2} \geq x$

31. $^{+}14 < x < {}^{+}25$

32. $^{-}6 < x \le {}^{+}7$

33. $0 \le x \le {}^{+}10$

34. $^{-}13 < x < {}^{-}8$

35. $^{+}1.5 < x \le {}^{+}5.5$

36. $\frac{1}{2} < x \le 2\frac{1}{2}$

37. $-3.25 \le x \le {}^{+}1.75$

38. $x \ne {}^{+}8$

39. $x \ne {}^{-}1$

40. $x \ne {}^{+}2.5$

41. $x \ne {}^{-}3\frac{1}{2}$

42. $x \ne {}^{-}11$

Directions: Explain each of the following graphs in words and write an inequality with *"x"* that represents it.

43.

< ================]

$^{-}6 \quad {}^{-}5 \quad {}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1$

44.

(==================================>

$^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1 \quad {}^{+}2 \quad {}^{+}3 \quad {}^{+}4 \quad {}^{+}5 \quad {}^{+}6 \quad {}^{+}7$

45.

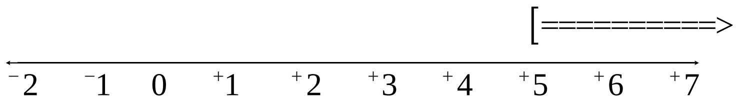
(===============================]

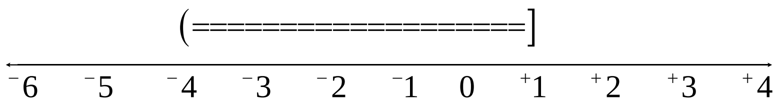
$^{-}6 \quad {}^{-}5 \quad {}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1 \quad {}^{+}2 \quad {}^{+}3 \quad {}^{+}4$

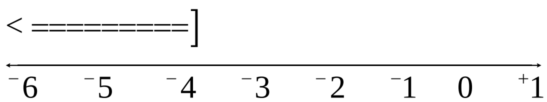
46.

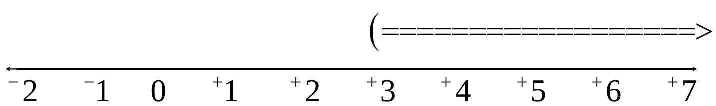
< ====================)

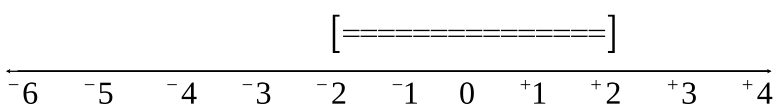
$^{-}6 \quad {}^{-}5 \quad {}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0 \quad {}^{+}1$

47. 

48. 

49. 

50. 

51. 

In statistics, we often have to compare two decimals and determine if one is larger than another. One common example of this is P-value. We often want to know if the P-value is less than or greater than 0.05.

52. Is a P-value of 0.035 less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

53. Is a P-value of 0.21 less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

54. Is a P-value of 0.0007 less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

55. Is a P-value of 1.48×10^{-6} less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

56. Is a P-value of 7.49×10^{-2} less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

57. $x - 3 < 5$

58. $12 \leq x - 5$

59. $m - 7 \leq -27$

60. $-4 > b - 1$

61. $-\frac{1}{4}n \leq 2$

62. $6 \leq \frac{-3}{5}w$

63. $\frac{x}{2} < -111$

64. $20 > -5c$

65. $-15c - 30 > 15$

66. $4x - x + 8 \leq 35$

67. $2x - 3 > 2(x - 5)$

68. $7x + 6 \leq 7(x - 4)$

69. $-6 < -3x < 15$

70. $-3 < 2x - 1 < 7$

Chapter 5 Review

In chapter 5, we looked at simplifying algebraic expressions by adding and subtracting like terms and by using the associative and distributive properties. Remember like terms have the exact same variable part. We cannot add *7f* to 5*w*. They are not like. If the terms are like, then we can add or subtract the numerical coefficients and keep the variable the same. ($8p-3p=5p$) We can always multiply terms by simply multiplying the numerical coefficients and putting the variables together $\left(7a\times 8b=56ab\right)$. When we want to multiply a number or term times a sum or difference, we need to use the distributive property like $3\left(4v+5w\right)=12v+15w$.

We also looked at solving equations. Remember the solution to an equation is the number or numbers that make the equation true. For example the solution to $3n+1=13$ is $n=4$ because when we plug in 4 for *n* we get 3(4)+1 = 12+1 = 13 (a true statement). Some equations have no solution and some equations have a solution of All Real Numbers.

The steps to solving equations are critical to remember. Here are the steps again in order. Remember that after each step, always add or subtract like terms that lie on the <u>same</u> side of the equation.

<u>Steps to Solving a Linear Equation</u>

1. Eliminate parenthesis by using the distributive property.
2. Eliminate fractions by multiplying both sides of the equation by the LCD.
3. Eliminate decimals by multiplying both sides of the equation by a power of 10 (10, 100, 1000…)
4. Use the addition property to eliminate variable terms so that there are only variables on one side of the equation.
5. Use the addition property to eliminate constants so that there are only constants on one side of the equation. The constants should be on the opposite side of the variables.
6. Use the multiplication property to multiply or divide both sides of the equation in order to isolate the variable by creating a coefficient of 1 for the variable.
7. Check your answer by plugging it into the original equation and see if the two sides are equal.

We looked at two equal fractions called a proportion. We say that we can solve a proportion by setting the cross products equal to each other and solving.

We also went over the meaning of the inequality symbols and learned to graph and understand them. Remember the following.

" < " means "less than". Notice the symbol looks like an arrow pointing to the left. For example $-5 < +9$ is true and $x < {}^{+}4$ means all real numbers less than 4. The graph would be a parenthesis at 4 shaded to the left.

" $\leq$ " means "less than or equal to". This symbol works like a "less than" with the added criteria that if the two numbers were equal it would still be true. For example: ${}^{-}7 \leq {}^{+}13$ is true and ${}^{+}19 \leq {}^{+}19$ is also true! (Remember that without the equal to part, ${}^{+}19 < {}^{+}19$ is <u>not</u> true.) $x \leq {}^{+}24$ means all real numbers less than or equal to 24. The graph would be a bracket at 24 shaded to the left.

" > " means "greater than". Notice the symbol looks like an arrow pointing to the right, For example ${}^{+}5 > {}^{+}1$ is true and $x > {}^{-}3$ means all real numbers greater than ${}^{-}3$. The graph would be a parenthesis at ${}^{-}3$ shaded to the right.

" $\geq$ " means "greater than or equal to". This symbol works like a "greater than" with the added criteria that if the two numbers were equal it would still be true. For example: ${}^{+}19 \geq {}^{+}6$ is true and ${}^{+}11 \geq {}^{+}11$ is also true! (Remember that without the equal to part, ${}^{+}11 > {}^{+}11$ is <u>not</u> true.) $x \geq {}^{+}15$ means all real numbers greater than or equal to 15. The graph would be a bracket at 15 shaded to the right.

" $\neq$ " means "not equal to". Notice the symbol looks like an equal sign with a line drawn through it. This symbol is only true if the two numbers are not equal. For example ${}^{+}1 \neq {}^{+}12$ is true but ${}^{-}8 \neq {}^{-}8$ is <u>not</u> true! Also $x \neq {}^{-}7$ represents all real numbers except ${}^{-}7$. To graph $x \neq {}^{-}7$ shade the entire number line and put a parenthesis at ${}^{-}7$.

Finally, we learned that to solve for an inequality, we will use the same steps to solving as we did equations, but if we have to multiply or divide both sides by a negative number, we must switch the sign.

<u>Chapter 5 Review Problems</u>

Simplify the following algebraic expressions. Tell how many terms the answer has and label it as monomial (1 term), binomial (2 terms), trinomial (3 terms) or multinomial (4 or more terms).

1. $-3c+7c-8c$

2. $6a-9b+{}^{-}11a-3b$

3. 7(5cd)

4. $\frac{1}{2}(6w)$

5. $-7(4x-9)$

6. $3a(b+4)$

7. $-5(2g+7)-3g+19$

8. $3h+19-(-2h+7)$

9. $-2(4q+7)-(3q-8)$

Solve the following equations. Simplify fraction answers completely.

10. $3x+6=12$

11. $9-4y=7$

12. $-3z+6=-4z-8$

13. $-7c+3+5c=2+3c+8$

14. $7(9a-2)=63a+8$

15. $-6(d-3)=d-7d+18$

16. $-\frac{1}{3}w+1=\frac{1}{2}w+\frac{1}{2}$

17. $\frac{1}{5}y-\frac{2}{3}=\frac{1}{3}y+\frac{1}{5}$

18. $-\frac{3}{4}p+2=-\frac{1}{2}p+\frac{5}{4}$

19. $-\frac{3}{5}v-\frac{1}{4}=\frac{3}{5}v-\frac{3}{4}$

20. $0.45x-0.9=0.35x+0.4$

21. $0.08\text{y}+0.012=-1.92y-0.034$

22. $0.05a+1.9=0.03x-0.4$

23. $1.5b+3=-2.5b-7$

24. $0.04(p+3)=0.15p+0.12-0.11p$

25. $\frac{2}{3}(2x+1)=\frac{1}{6}x$

Solve the following proportions. Simplify fraction answers completely.

26. $\frac{-4}{w}=\frac{3}{8}$

27. $\frac{7}{9}=\frac{x+1}{18}$

28. $\frac{3}{F+4}=\frac{5}{F-1}$

29. $\frac{6}{-7}=\frac{g+4}{2}$

30. $\frac{-2w-1}{5}=\frac{w+1}{4}$

Determine if the inequality symbol is used correctly or incorrectly?

31. ${}^{+}3 > {}^{+}19$

32. ${}^{-}13 \geq {}^{+}3$

33. ${}^{-}23 < {}^{-}10 \leq {}^{+}1$

34. ${}^{+}11 \leq {}^{+}11$

35. ${}^{+}1.5 \neq {}^{+}1\frac{1}{2}$

36. ${}^{-}5 < {}^{-}7 \leq 0$

Graph the following inequalities on the number line.

37. $x \leq {}^{-}6$

38. $x > {}^{+}4\frac{1}{2}$

39. ${}^{-}7 \geq x$

40. ${}^{+}1.75 \leq x$

41. ${}^{-}5 < x \leq {}^{+}8$

42. ${}^{-}5 \leq x \leq 0$

43. ${}^{+}15 < x < {}^{+}18$

44. ${}^{-}3.5 < x \leq {}^{+}4.5$

45. $x \neq {}^{+}9$

46. $x \neq 0$

For each of the following graphs, write an inequality with *"x"* that represents it.

47.

```
< ======================]
⁻6   ⁻5   ⁻4   ⁻3   ⁻2   ⁻1   0   ⁺1
```

48.

```
              (==========================>
⁻2   ⁻1   0   ⁺1   ⁺2   ⁺3   ⁺4   ⁺5   ⁺6   ⁺7
```

49.

```
                          (===============]
⁻6   ⁻5   ⁻4   ⁻3   ⁻2   ⁻1   0   ⁺1   ⁺2   ⁺3   ⁺4
```

50.

```
               [========)
⁻6   ⁻5   ⁻4   ⁻3   ⁻2   ⁻1   0   ⁺1   ⁺2   ⁺3   ⁺4
```

51.

```
<============================)(============>
⁻6   ⁻5   ⁻4   ⁻3   ⁻2   ⁻1   0   ⁺1   ⁺2   ⁺3   ⁺4
```

52. Is a P-value of 0.0843 less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

53. Is a P-value of 6.24×10^{-5} less or greater than 0.05? Use an inequality symbol to show your answer symbolically.

Solve the following inequalities for the variable.

54. $2x - 1 > 5$

55. $3n + 1 \leq 13$

56. $-4x + 5 < 1$

57. $-2x + 7 > 3$

58. $2d + 5 \leq -1$

59. $-2d + 5 \leq 5$

60. $-1 < 3x + 2 < -7$