Chapter 3 Review Sheet with Answers

Topics:

- Use population claims to construct the null and alternative hypothesis. Also, know how to determine the type of tail for the test.
- Know the assumptions (conditions) necessary to do a one-population mean or proportion hypothesis test.
- Be able to explain the meaning of the Z-test statistic for one-population proportion hypothesis tests.
- Be able to explain the meaning of the T-test statistic for one-population mean average hypothesis tests.
- Be able to determine how likely it is for the sample data to have occurred by sampling variability (random chance).
- Know how to interpret significance by judging whether the test statistic falls in the tail corresponding to the critical value.
- Know how to interpret significance by judging whether the sample statistic falls in the tail corresponding to the significance level in a randomized simulation.
- Know how to use "Theoretical Distributions" menu in StatKey to use the significance level to look up critical values.
- Know how to use "Theoretical Distributions" menu in StatKey to use the test statistic to look up the P-value.
- Know how to create randomized simulations and use the significance level to judge the tails.
- Know how to create randomized simulations with StatKey and use the sample statistic to calculate the P-value.
- Know how to use the P-value and significance level to determine if the sample data could have happened because of sampling variability or if it was unlikely.
- Know the definition of P-value.
- Know how to use the P-value and significance level to determine if we should reject the null hypothesis or fail to reject the null hypothesis.
- Be able to write the formal conclusion for a hypothesis test and explain its meaning.
- Know the definitions and terms associated with Type 1 and Type 2 errors. Know how to decrease the chances of having a Type 1 or Type 2 error.

Chapter 3 Vocabulary Terms and Definitions

Population: The collection of all people or objects to be studied.

Sample: Collecting data from a small subgroup of the population.

Random Sample: Sample data collected in such a way that everyone in the population has an equal chance to be included.

Statistic: A number calculated from sample data in order to understand the characteristics of the data. For example, a sample mean average, a sample standard deviation, or a sample percentage.

Parameter: A number that describes the characteristics of a population like a population mean or a population percentage. Can be calculated from an unbiased census, but is often just a guess about the population.

Hypothesis Test: A procedure for testing a claim about a population.

Null Hypothesis (H_0): A statement about the population that involves equality. It is often a statement about "no change", "no relationship" or "no effect".

Alternative Hypothesis (H_A or H_1): A statement about the population that does not involve equality. It is often a statement about a "significant difference", "significant change", "relationship" or "effect".

Population Claim: What someone thinks is true about a population.

Test Statistic: A number calculated in order to determine if the sample data significantly disagrees with the null hypothesis. There are a variety of different test statistics depending on the type of data.

One-Population Proportion Test Statistic (z): The sample proportion is this many standard errors above or below the population proportion in the null hypothesis.

One-Population Mean Test Statistic (t): The sample mean is this many standard errors above or below the population mean in the null hypothesis.

Critical Value: A number we compare our test statistic to in order to determine significance. In a sampling distribution or a theoretical distribution approximating the sampling distribution, the critical value shows us where the tail or tails are. The test statistic must fall in the tail to be significant.

Sampling Variability: Also called "random chance". The principle that random samples from the same population will usually be different and give very different statistics. The random samples will usually be different than the population parameter.

P-value: The probability of getting the sample data or more extreme because of sampling variability (by random chance) if the null hypothesis is true.

Significance Level (α): Also called the Alpha Level. This is the probability of making a type 1 error. The P-value is compared to this number to determine significance and sampling variability. If the P-value is lower than the significance level, then the sample data significantly disagrees with the null hypothesis and is unlikely to have happened because of sampling variability.

Randomized Simulation: A technique for visualizing sampling variability in a hypothesis test. The computer assumes the null hypothesis is true, and then generates random samples. If the sample data or test statistic falls in the tail, then the sample data significantly disagrees with the null hypothesis. This technique can also calculate the P-value and standard error without a formula.

Type 1 Error: When biased sample data leads you to support the alternative hypothesis when the alternative hypothesis is actually wrong in the population.

Type 2 Error: When biased sample data leads you fail to reject the null hypothesis when the null hypothesis is actually wrong in the population.

Beta Level (β): The probability of making a type 2 error.

Conclusion: A final statement in a hypothesis test that addresses the claim and evidence.

Hypothesis Test Notes

Steps to Writing the Null and Alternative Hypothesis

1. **Write down the population claim in symbolic notation.**
   (Note: This can be the null or the alternative hypothesis.)
   (Note: This is the statement the scientist thinks is true.)
   (Note: In one-population tests, always write the parameter on the left and the number on the right.)

2. **Write down the opposite of the population claim.**
   (Note: This can be the null or the alternative hypothesis.)
   (Note: This is the statement that would be true if the scientist is wrong.)
   (Note: In one-population tests, always write the parameter on the left and the number on the right.)

3. **The statement with equality $(=, \leq, \geq)$ is always the null hypothesis (H_0).**
   (Note: The null hypothesis is usually " = ". Statements with " ≤ " or " ≥ " are often changed to " = ".)

4. **The statement that does NOT have equality $(\neq, <, >)$ is always the alternative hypothesis (H_A).**

Which Tail should we use in a one-population Hypothesis Test?

- **Right-Tailed Test: Alternative Hypothesis (H_A) is GREATER THAN (>)**
- **Left-Tailed Test: Alternative Hypothesis (H_A) is LESS THAN (<)**
- **Two-Tailed Test: Alternative Hypothesis (H_A) is NOT EQUAL (≠)**

One-Population Hypothesis Test Assumptions (Conditions)

Assumptions for One-Population Mean Hypothesis Test

- **Random Sample Data**
- **Individuals Independent**
- **Sample data normal or sample size at least 30**

Assumptions for One-Population Proportion (%) Hypothesis Test

- **Random Sample Data**
- **Individuals Independent**
- **Sample data has at least 10 successes and at least 10 failures.**

Assumptions for One-Population Randomized Simulation Hypothesis Test

- **Random Sample Data**
- **Individuals Independent**

Summary Table of P-Value, Test Statistic, Simulation, Significance, Sampling Variability and Evidence

		SIGNIFICANT		NOT SIGNIFICANT
		Test Statistic falls in tail determined by a critical value		**Test Statistic does NOT fall in tail determined by a critical value**
		OR		*OR*
		Low P-value *(P-value ≤ significance level)*		**High P-value** *(P-value > significance level)*
		OR		*OR*
		Sample Statistic falls in tail determined by the significance level of randomized simulation.		**Sample Statistic does NOT fall in tail determined by the significance level of randomized simulation.**
Is the sample data significantly different than H_0?		**Significantly Different**		**NOT Significantly Different**
Could the sample data happen because of sampling variability (random chance) if H_0 is true?		**Unlikely**		**Could Happen**
Reject H_0 or Fail to Reject H_0?		**Reject H_0**		**Fail to Reject H_0**
Is there significant evidence?		**Significant Evidence**		**NOT Significant Evidence**

Summary Table: Conclusions

	Claim is H_0	Claim is H_A
Low P-value *(P-value ≤ significance level)*	**There is significant evidence to reject the claim.** *(Evidence indicates that the claim may be wrong.)*	**There is significant evidence to support the claim.** *(Evidence indicates that the claim may be correct.)*
High P-value *(P-value > significance level)*	**There is NOT significant evidence to reject the claim.** *(Random sample data does not disagree with the claim. The claim could be true, but we do not have evidence.)*	**There is NOT significant evidence to support the claim.** *(Random sample data disagrees with the claim. The claim could be wrong, but we do not have evidence.)*

Chapter 3 Review Practice Problems

1. Write a definition for the following key terms.
   - hypothesis test
   - Null hypothesis
   - Alternative Hypothesis
   - Population Claim
   - Test statistic
   - one-population proportion Z test statistic
   - one-population mean T test statistic
   - Critical Value
   - Sampling Variability (Random Chance)
   - P-value
   - significance level (alpha level)
   - Randomized Simulation
   - beta level
   - type 1 error
   - type 2 error
   - Conclusion

2. How is randomized simulation used in hypothesis testing and describe what it can tell us.

3. How can we know if the sample data significantly disagrees with the null hypothesis?

4. How can we determine how likely it is for the sample data to have occurred because of sampling variability?

5. How do we know if we reject the null hypothesis or fail to reject the null hypothesis?

6. What are the four steps to writing conclusions?

7. What assumptions do we need to check for the following:

   When testing a hypothesis about a one-population mean average?
   When testing a hypothesis about a one-population proportion (percentage)?
   When testing a hypothesis about one-population using randomized simulation?

8. Fill out the following table regarding test statistics and critical values.

Test Statistic	Critical Value	Does sample significantly disagree with H_0 or not?
T = +1.774	±2.751	
Z = −2.481	−1.96	
T = −3.394	±2.566	
Z = +1.362	+1.645	

9. Fill out the following table regarding P-value and Significance levels. Assume the data passed conditions.

P-value	P-value %	Significance Level	Sampling Variability or Unlikely	Reject H_0 or Fail to reject H_0?
0.0002		5%		
0.3327		1%		
1.84×10^{-5}		10%		
0.0941		5%		

10. Fill out the following table to practice writing conclusions. Assume the data passed conditions.

P-value	Claim	Write the Conclusion addressing Evidence and claim
Low	H_0	
High	H_A	
High	H_0	
Low	H_A	

11. Write the null and alternative hypotheses for the following. Which is the claim? Is this a left-tailed, right-tailed, or two-tailed test?

a) "We used to think that the population mean average normal body temperature for all humans was exactly 98.6 degrees Fahrenheit. Now we think it is lower."

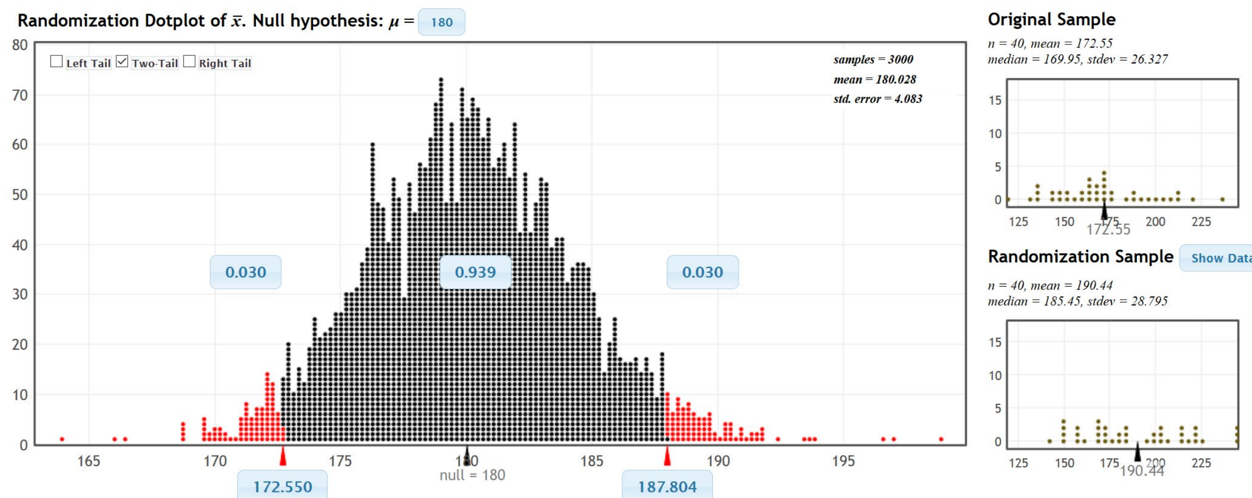
b) "The percentage of all people in China with Typhoid (π_1) used to be higher than the percentage of all people in India with Typhoid (π_2). Now we think the percentage is about the same."

c) "The population mean average age of students at UCLA (μ_1) is the same as the population mean average age of students at USC (μ_2)."

12. Answer the following questions about type 1 and type 2 errors.

a) What is a type 1 error?
b) What is a type 2 error?
c) What is the probability of type 1 error called?
d) What is the probability of type 2 error called?
e) Why do type 1 and type 2 errors occur?
f) What can we do to limit the chances of a type 1 error?
g) What can we do to limit the chances of a type 2 error?
h) Which significance level is best for keeping both type 1 and type 2 errors low?
i) If the significance level is 1%, what happens to the probability of type 1 and type 2 errors?
j) If the significance level is 10%, what happens to the probability of type 1 and type 2 errors?

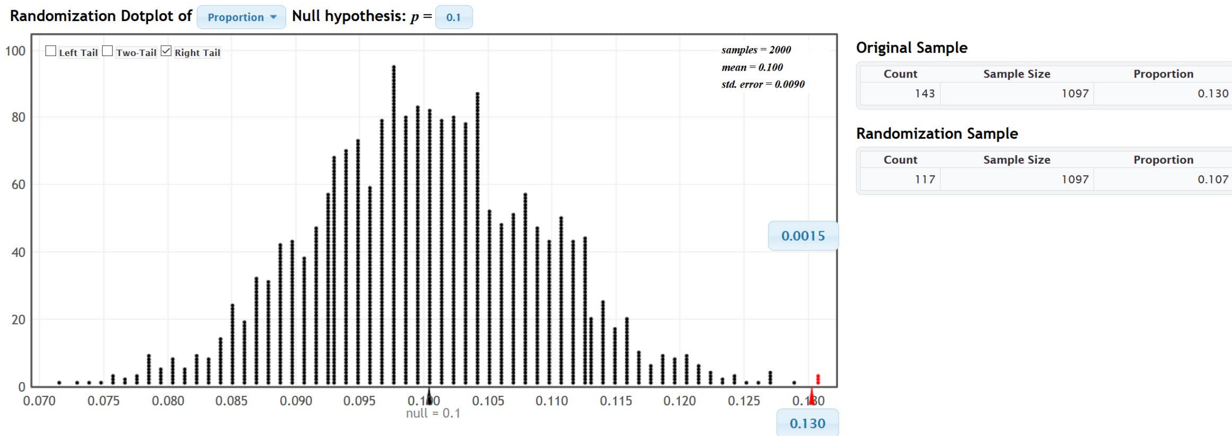
13. An article states that the mean average weight of men in the U.S. is 180 pounds. The random health data at teachoutcoc.org was pasted into StatKey and the following randomized simulation was created in order to test the article's claim. Use a 1% significance level. Make sure to check the assumptions. Give the null and alternative hypothesis, estimate the P-value, and the conclusion. Write a sentence to explain the P-value. Was there a significant difference between the sample mean and the population mean? How likely was it that the sample data occurred by random chance if the population mean really is 180 pounds?



14. Use the Statcato printout below to test the following claim. A speaker at a nursing convention said that the population mean average hourly salary of a registered nurse used to be $25 per hour, but now is greater than $25 per hour. A random sample of 28 nurses gave a sample mean of $26.82 and a standard deviation of $4.37 . A histogram of the salaries showed a bell shaped distribution. Use a 5% significance level. Make sure to check the assumptions. Give the null and alternative hypothesis, test statistic, P-value, and the conclusion. Write a sentence to explain the test statistic. Write a sentence to explain the P-value. Was there a significant difference between the sample mean and the population mean? How likely was it that the sample data occurred by random chance if the population mean really is $25?

N	Sample Mean	Stdev s	Significance Level	Critical Value	Test Statistic	p-Value
28	26.82	4.37	0.05	1.703	2.204	0.0181

15. Use simulation on www.lock5stat.com to test the following claim. The Harris Poll conducted a random survey in which they asked 1097 women "How many tattoos do you currently have?" Of the 1,097 females surveyed, 143 responded that they had at least one tattoo. A tattoo magazine claimed that more than 10% of women have at least one tattoo. Use a 5% significance level to test the magazine's claim. Give the null and alternative hypothesis, the estimated P-value, and the conclusion. Write a sentence to explain the P-value. Was there a significant difference between the sample percent and the population value? How likely was it that the sample data occurred by random chance from a 10% population.



Answers to Chapter 3 Review Problems

1.

Hypothesis Test: A procedure for testing a claim about a population.

Null Hypothesis (H_0): A statement about the population that involves equality. It is often a statement about "no change", "no relationship" or "no effect".

Alternative Hypothesis $(H_A \; or \; H_1)$: A statement about the population that does not involve equality. It is often a statement about a "significant difference", "significant change", "relationship" or "effect".

Population Claim: What someone thinks is true about a population.

Test Statistic: A number calculated in order to determine if the sample data significantly disagrees with the null hypothesis. There are a variety of different test statistics depending on the type of data.

One-Population Proportion Test Statistic (z): The sample proportion is this many standard errors above or below the population proportion in the null hypothesis.

One-Population Mean Test Statistic (t): The sample mean is this many standard errors above or below the population mean in the null hypothesis.

Critical Value: A number we compare our test statistic to in order to determine significance. In a sampling distribution or a theoretical distribution approximating the sampling distribution, the critical value shows us where the tail or tails are. The test statistic must fall in the tail to be significant.

Sampling Variability: Also called "random chance". The principle that random samples from the same population will usually be different and give very different statistics. The random samples will usually be different than the population parameter.

P-value: The probability of getting the sample data or more extreme because of sampling variability (by random chance) if the null hypothesis is true.

Significance Level (α): Also called the Alpha Level. This is the probability of making a type 1 error. The P-value is compared to this number to determine significance and sampling variability. If the P-value is lower than the significance level, then the sample data significantly disagrees with the null hypothesis and is unlikely to have happened because of sampling variability.

Randomized Simulation: A technique for visualizing sampling variability in a hypothesis test. The computer assumes the null hypothesis is true, and then generates random samples. If the sample data or test statistic falls in the tail, then the sample data significantly disagrees with the null hypothesis. This technique can also calculate the P-value and standard error without a formula.

Type 1 Error: When biased sample data leads you to support the alternative hypothesis when the alternative hypothesis is actually wrong in the population.

Type 2 Error: When biased sample data leads you fail to reject the null hypothesis when the null hypothesis is actually wrong in the population.

Beta Level (β): The probability of making a type 2 error.

Conclusion: A final statement in a hypothesis test that addresses the claim and evidence.

2.

Randomized simulation is used to determine if sample data significantly disagrees with the null hypothesis and if the sample data occurred by random chance. The simulation can be used to calculate the P-value and determine the tail or tails and significance without a formula, test statistic, critical value, or theoretical curve. It also has less assumptions than traditional formula hypothesis tests.

3.

We can determine if sample data significantly disagrees with the null hypothesis in three ways.

If the test statistic falls in a tail determined by the critical value.

If the P-value is lower than the significance level.

If the sample statistic falls in a tail of a randomized simulation with the tail proportion being the significance level.

4.

Calculate the P-value. The P-value determines the probability of the sample statistic (sample data) or more extreme occurring by sampling variability if the null hypothesis was true.

5.

If the P-value is less than or equal to the significance level and the data passes conditions, we reject the null hypothesis.

If the P-value is greater than the significance level, we will fail to reject the null hypothesis.

If the P-value is less than or equal to the significance level but the data fails on or more condition, we fail to reject the null hypothesis.

6.

If the P-value is low and data passes conditions, start the conclusion with “there is significant evidence”.

If the P-value is high or data did not pass all conditions, start the conclusion with “there is not significant evidence”.

If the claim is the null hypothesis, finish the conclusion with “to reject the claim”.

If the claim is the alternative hypothesis, finish the conclusion with “to support the claim”.

7.

One-population Mean Assumptions

- The quantitative sample data should be collected randomly or be representative of the population.
- Data values within the sample should be independent of each other.
- The sample size should be at least 30 or have a nearly normal shape.

One-population Proportion Assumptions

- The categorical sample data should be collected randomly or be representative of the population.
- Data values within the sample should be independent of each other.
- There should be at least ten successes and at least ten failures.

One-Population Randomized Simulation Assumptions

- The sample data should be collected randomly or be representative of the population.
- Data values within the sample should be independent of each other.

8. Fill out the following table regarding test statistics and critical values.

Test Statistic	Critical Value	Does sample significantly disagree with H_0 or not?
T = +1.774	±2.751	Does not significantly disagree since test stat not in tail.
Z = −2.481	−1.96	Does significantly disagree since test stat in tail.
T = −3.394	±2.566	Does significantly disagree since test stat in tail.
Z = +1.362	+1.645	Does not significantly disagree since test stat not in tail.

9. Fill out the following table regarding P-value and Significance levels. Assume the data passed conditions.

P-value	P-value %	Significance Level	Sampling Variability or Unlikely	Reject H_0 or Fail to reject H_0?
0.0002	0.02%	5%	Unlikely since P-value low	Reject Ho
0.3327	33.27%	1%	Could be sampling variability since P-value High	Fail to reject Ho
1.84×10^{-5}	0.00184%	10%	Unlikely since P-value low	Reject Ho
0.0941	9.41%	5%	Could be sampling variability since P-value High	Fail to reject Ho

10. Fill out the following table to practice writing conclusions. Assume data passed the conditions.

P-value	Claim	Write the Conclusion addressing Evidence and claim
Low	H_0	There is significant evidence to reject the claim.
High	H_A	There is not significant evidence to support the claim.
High	H_0	There is not significant evidence to reject the claim.
Low	H_A	There is significant evidence to support the claim.

11.

a.

H_0: μ = 98.6°F

H_A: μ < 98.6°F (Claim)

Left-tailed test

b.

H_0: π_1 = π_2 (Claim)

H_A: π_1 > π_2

Right-tailed test

c.

H_0: μ_1 = μ_2 (Claim)

H_A: $\mu_1 \neq \mu_2$

Two-tailed test

12.

a.

A Type 1 Error occurs when biased sample data gives you a low P-value and leads you to reject the null hypothesis and support the alternative hypothesis, when the alternative hypothesis is actually wrong in the population.

b.

A Type 2 Error occurs when biased sample data gives you a high P-value and leads you to fail to reject the null hypothesis when the null hypothesis is actually wrong in the population.

c.

Alpha Level (α) or Significance Level

d.

Beta Level (β)

e.

Any time sample data does not reflect the population a type 1 or type 2 error may occur. It is often due to poor sampling techniques, not recognizing bias, or just sampling variability.

f.

To limit the chances of a type 1 error, decrease the significance level (alpha level).

g.

There are two ways to limit the chances of type 2 error. The preferred method is to raise the sample size (collect more data). If that is not possible, you can also raise the significance level.

h.

A 5% significance level tends to keep both type 1 and type 2 errors low.

i.

At a 1% significance level, there is a lower probability of type 1 error and a higher probability of type 2 error.

j.

At a 10% significance level, there is a higher probability of type 1 error and a lower probability of type 2 error.

13.

H_0: μ = 180 pounds (claim)

H_A: μ ≠ 180 pounds

Two-tailed test

Assumptions Check

Random Sample? Yes. Given in the problem.

Individuals independent? Yes. This is a small random sample out of a huge population. The men are not likely to be related.

P-value (Two-tailed) = 0.030 + 0.030 = 0.060 = 6.0%

If the null hypothesis is true and the population mean average weight of all men is 180 pounds, then there is 6.0% probability of getting this sample data or more extreme by random chance.

Since the P-value is higher than our significance level, the sample data does not significantly disagree with the null hypothesis and could happen by random chance.

Fail to reject Ho.

There is not significant evidence to reject the articles claim that the population mean average weight of all men is 180 pounds.

(The article could be correct. We do not have any evidence to contradict it.)

14.

H_0: μ = $25

H_A: μ > $25 (claim)

Right-tailed test

Assumptions Check

Random Sample? Yes. Given in the problem.

Individuals independent? Yes. This is a small random sample out of a huge population. The men are not likely to be related.

At least 30 or normal? The sample size is below 30, but since the histogram showed a normal shape, the data does pass the "30 or normal" requirement.

Test Statistic = 2.204

The sample mean of $26.82 is 2.204 standard errors above the population mean of $25. The test statistic indicates that the sample data significantly disagrees with the null hypothesis since it falls in the tail determined by the critical value.

P-value = 0.0181 = 1.81%

If the null hypothesis is true and the population mean average salary of nurses is $25, then there is 1.81% probability of getting this sample data or more extreme by random chance.

Since the P-value is lower than our significance level, the sample data was unlikely to happen by random chance.

Reject Ho.

There is significant evidence to support the claim that that the population mean average salary for registered nurses is above $25.

(The speaker at the convention is probably correct and we have evidence to back them up.)

15.

H_0: π = 0.1

H_A: π > 0.1 (claim)

Right-tailed test

Assumptions Check

Random Sample? Yes. Given in the problem.

Individuals independent? Yes. This is a small random sample out of a huge population. The women are not likely to be related.

P-value = 0.0015 = 0.15%

If the null hypothesis is true and the population proportion of women with a tattoo is 10%, then there is 0.15% probability of getting this sample data or more extreme by random chance.

Since the P-value is lower than our significance level, the sample data does significantly disagree with the null hypothesis and is unlikely to happen by random chance.

Reject Ho.

There is significant evidence to support the claim that more than 10% of all women have at least one tattoo.

(The article is probably correct and we have evidence to back it up.)
