

Stat Support Activity: Calculating Matched Pair Mean Difference Confidence Intervals

Notes

- Suppose we want to compare the population means using two bits of quantitative data taken from the same object or person. We would use a matched pair mean difference confidence interval. Instead of calculating sample means from two data sets and subtracting them, we would subtract each individual pair of data values. The resulting column is called the difference column (d). We can then use a one-population confidence interval formula on the difference column.

\bar{d} = Sample mean average of the difference column.

s_d = Sample standard deviation of the difference column.

n = sample size (In this case the number of ordered pairs.)

$df = n - 1$ (degrees of freedom)

T_c = Critical Value T-score corresponding to degrees of freedom and confidence level.

- Standard Error = $\frac{s_d}{\sqrt{n}}$
- Margin of Error = $T_c \times \frac{s_d}{\sqrt{n}}$
- Matched Pair Confidence Interval Formula

$$\bar{d} \pm \left(T_c \times \frac{s_d}{\sqrt{n}} \right)$$

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Problem Directions: Use the provided statistics to answer the following questions and calculate the matched pair two-population mean confidence interval.

1.

The ACT exam is used by many colleges to test the readiness of high school students for college. Many high school students are now taking ACT prep classes. A local high school offers an ACT prep class, but wants to know if it really helps. Twenty-eight students were randomly selected. They took the ACT exam before and after taking the ACT prep class. Population 1 is the ACT scores after taking the prep class and population 2 is the ACT scores before taking the prep class. The sample mean of the differences was 5.8 ACT points and the sample standard deviation of the differences was 4.3 ACT points.

Difference = After Prep Class ACT Score – Before Prep Class ACT Score

$\bar{d} = 5.8$ ACT points

$s_d = 4.3$ ACT points

$n = 28$ ordered pair differences

- What is the degrees of freedom?
- Go to www.lock5stat.com. Under the “theoretical distributions” area click on “T”. Use a 90% confidence level and the degrees of freedom to look up the critical value T-score (T_c).
- Calculate the standard error using the formula.

$$\text{Standard Error} = \frac{s_d}{\sqrt{n}} = ???$$

- Calculate the margin of error by multiplying the answers to part (b) and part (c).

$$\text{Margin of Error} = T_c \times \frac{s_d}{\sqrt{n}} = ???$$

- Calculate the lower limit of the confidence interval.

$$\text{Lower Limit} = \bar{d} - \text{Margin of Error}$$

- Calculate the upper limit of the confidence interval.

$$\text{Upper Limit} = \bar{d} + \text{Margin of Error}$$

- Were the ACT scores after the prep class significantly better than the ACT scores before the prep class? Explain how you know.

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2.

We want to compare the population mean average diastolic and systolic blood pressures for 40 randomly selected women. Population 1 was diastolic blood pressure and population 2 was systolic blood pressure.

Difference = Diastolic Blood Pressure – Systolic Blood Pressure

$$\bar{d} = -43.375 \text{ millimeters of Mercury}$$

$$s_d = 10.748 \text{ millimeters of Mercury}$$

$n = 40$ ordered pair differences

- What is the degrees of freedom?
- Go to www.lock5stat.com. Under the “theoretical distributions” area click on “T”. Use a 95% confidence level and the degrees of freedom to look up the critical value T-score (T_c).
- Calculate the standard error using the formula.

$$\text{Standard Error} = \frac{s_d}{\sqrt{n}} = ???$$

- Calculate the margin of error by multiplying the answers to part (b) and part (c).

$$\text{Margin of Error} = T_c \times \frac{s_d}{\sqrt{n}} = ???$$

- Calculate the lower limit of the confidence interval.

$$\text{Lower Limit} = \bar{d} - \text{Margin of Error}$$

- Calculate the upper limit of the confidence interval.

$$\text{Upper Limit} = \bar{d} + \text{Margin of Error}$$

- Does the confidence interval confirm that diastolic blood pressure for women is significantly lower than systolic blood pressure? Explain how you know.
- How much lower could women’s diastolic blood pressure be than their systolic blood pressure?

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3.

A random sample of 53 people living in L.A. were asked the average amount they spend on groceries in dollars per month in 2022 and in 2023. We want to compare the population mean average spent in 2023 (population 1) and the population mean average spent in 2022 (population 2).

Differences = \$ spent on groceries in 2023 - \$ spent on groceries in 2022

$\bar{d} = -23.88$ dollars per month

$s_d = 174.22$ dollars per month

$n = 53$ ordered pair differences

- What is the degrees of freedom?
- Go to www.lock5stat.com. Under the “theoretical distributions” area click on “T”. Use a 99% confidence level and the degrees of freedom to look up the critical value T-score (T_c).
- Calculate the standard error using the formula.

$$\text{Standard Error} = \frac{s_d}{\sqrt{n}} = ???$$

- Calculate the margin of error by multiplying the answers to part (b) and part (c).

$$\text{Margin of Error} = T_c \times \frac{s_d}{\sqrt{n}} = ???$$

- Calculate the lower limit of the confidence interval.

$$\text{Lower Limit} = \bar{d} - \text{Margin of Error}$$

- Calculate the upper limit of the confidence interval.

$$\text{Upper Limit} = \bar{d} + \text{Margin of Error}$$

- Is the money spent on groceries in L.A. per month in 2023 significantly different than in 2022? Explain how you know.