# Stat Support Activity: ANOVA Calculating F Test Statistic

**ANOVA Notes:**

- **ANOVA stands for Analysis of Variance.**
- **ANOVA test is used to determine if a categorical variable is related or not related to a quantitative variable.**
- **Multiple mean average test. Compares quantitative data sample mean averages for each group created from the categorical data.**
- **For an ANOVA test, we usually collect a random sample of ordered pair data, with a categorical response and a quantitative response.**
- **ANOVA is always a right-tailed test that uses the F-test statistic and the F-distribution.**
- **The ANOVA test and the F-test statistic were invented by the famous statistician R.A. Fisher. It was originally called "Fisher's test statistic", but was later shortened to "F – test statistic".**

**F – Test Statistic Notes**

- **n = total sample size (total number of people or objects)**
- **n – 1 = total degrees of freedom**
- **k = number of groups created by categorical data.**
- **k – 1 = degrees of freedom between the groups. (Also called the degrees of freedom groups or the degrees of freedom for numerator.)**
- **(n – 1) – (k – 1) = n – k = degrees of freedom within the groups. (Also called the degrees of freedom error or the degrees of freedom for the denominator.)**
- $\bar{\bar{x}}$ **= mean of all of the quantitative data. Also called the "mean of means".**
- **Sum of Squares between the groups (SS between or SS groups) =** $\sum(\bar{x}_i - \bar{\bar{x}})^2$ *(Subtract sample mean for each group minus the mean of means (means of all people or objects). Then square each difference calculated and add up all the squares.)*
- **Sum of Squares within the groups (SS error or SS within) =** $\sum(x - \bar{x}_i)^2$ *(Subtract each data value minus the sample mean of its group. Then square each difference and add up all the squares.)*
- **Variance Between the Groups ("MS between" or "MS groups")**
  **=** $\frac{Sum\ of\ Squares\ Between}{Degrees\ of\ Freedom\ Between} = \frac{\sum(\bar{x}_i - \bar{\bar{x}})^2}{k-1}$
- **Variance Within the Groups ("MS within" or "MS error")**
  **=** $\frac{Sum\ of\ Squares\ Within}{Degrees\ of\ Freedom\ Within} = \frac{\sum(x - \bar{x}_i)^2}{n-k}$
- **F – test statistic =** $\frac{Variance\ Between\ the\ Groups}{Variance\ Within\ the\ Groups} = \frac{\left(\frac{Sum\ of\ Squares\ Between}{Degrees\ of\ Freedom\ Between}\right)}{\left(\frac{Sum\ of\ Squares\ Within}{Degrees\ of\ Freedom\ Within}\right)}$
- **The F – test statistic is significant if it falls in the right tail of the F – distribution corresponding to the critical value. This would mean that the variance between is significantly larger than the variance within. If the data is unbiased and meets conditions, then the categorical and quantitative variables are probably related.**
- **The F – test statistic is NOT significant if it does NOT fall in the right tail of the F – distribution corresponding to the critical value. This would mean that the variance between is NOT significantly larger than the variance within and that the categorical and quantitative data are probably not related.**

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**Problems**

**1.**

Total Sample Size (n) = 24

Number of Groups (k) = 3

Sum of Squares Between = 1561

Sum of Squares Within = 2913

Sum of Squares Total = 4474

a) Degrees of Freedom Between the groups = k – 1 = ???

b) Degrees of Freedom Within the groups = n – k = ???

c) Total Degrees of freedom = n – 1 = ???

d) Add the Degrees of Freedom Between to the Degrees of Freedom Within. Do they add up to the Total Degrees of Freedom (yes or no)?

e) Add the Sum of Squares Between to the Sum of Squares Within. Do they add up to the Total Sum of Squares (yes or no)?

f) Variance Between = $\frac{Sum\ of\ Squares\ Between}{Degrees\ of\ Freedom\ Between}$ = ???

g) Variance Within = $\frac{Sum\ of\ Squares\ Within}{Degrees\ of\ Freedom\ Within}$ = ???

h) F – Test Statistic = $\frac{Variance\ Between\ the\ Groups}{Variance\ Within\ the\ Groups}$ = ???

*i)* F – Test Statistic Sentence: The ratio of the variance between the groups to the variance within the groups is _____. *(Note: If the F -test stat is greater than 1, this also indicates that the Variance Between is _____ times larger than the Variance Within.)*

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**2.**

Total Sample Size (n) = 451

Number of Groups (k) = 4

Sum of Squares Between = 5926.3

Sum of Squares Within = 41023.8

Sum of Squares Total = 46950.1

a) Degrees of Freedom Between the groups = k – 1 = ???

b) Degrees of Freedom Within the groups = n – k = ???

c) Total Degrees of freedom = n – 1 = ???

d) Add the Degrees of Freedom Between to the Degrees of Freedom Within. Do they add up to the Total Degrees of Freedom (yes or no)?

e) Add the Sum of Squares Between to the Sum of Squares Within. Do they add up to the Total Sum of Squares (yes or no)?

f) Variance Between = $\frac{Sum\ of\ Squares\ Between}{Degrees\ of\ Freedom\ Between}$ = ???

g) Variance Within = $\frac{Sum\ of\ Squares\ Within}{Degrees\ of\ Freedom\ Within}$ = ???

h) F – Test Statistic = $\frac{Variance\ Between\ the\ Groups}{Variance\ Within\ the\ Groups}$ = ???

*i)* F – Test Statistic Sentence: The ratio of the variance between the groups to the variance within the groups is _____. *(Note: If the F -test stat is greater than 1, this also indicates that the Variance Between is _____ times larger than the Variance Within.)*

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**3.**

Total Sample Size (n) = 175

Number of Groups (k) = 5

Sum of Squares Between = 10484530.0

Sum of Squares Within = 56249274.8

Sum of Squares Total = 66733804.8

a) Degrees of Freedom Between the groups = k – 1 = ???

b) Degrees of Freedom Within the groups = n – k = ???

c) Total Degrees of freedom = n – 1 = ???

d) Add the Degrees of Freedom Between to the Degrees of Freedom Within. Do they add up to the Total Degrees of Freedom (yes or no)?

e) Add the Sum of Squares Between to the Sum of Squares Within. Do they add up to the Total Sum of Squares (yes or no)?

f) Variance Between = $\frac{Sum\ of\ Squares\ Between}{Degrees\ of\ Freedom\ Between}$ = ???

g) Variance Within = $\frac{Sum\ of\ Squares\ Within}{Degrees\ of\ Freedom\ Within}$ = ???

h) F – Test Statistic = $\frac{Variance\ Between\ the\ Groups}{Variance\ Within\ the\ Groups}$ = ???

*i)* F – Test Statistic Sentence: The ratio of the variance between the groups to the variance within the groups is _____. *(Note: If the F -test stat is greater than 1, this also indicates that the Variance Between is _____ times larger than the Variance Within.)*