

**Practice Problems Section 4A (Updated Version)**

**(#1-10) Use each of the following Two population mean and Matched Pair T-test statistics and the corresponding critical values to fill out the table.**

	Type of Test	T-test stat	Sentence to explain T-test statistic.	Critical Value	Does the T-test statistic fall in a tail determined by a critical value or not?	Are the sample means from the two independent groups significantly different or not? (If matched pair, is the sample mean of differences significantly different than zero? Explain.	Does sample mean difference significantly disagree with $H_0$ ? Explain.
1.	Right Tailed Indep. Groups	+1.383		+2.447			
2.	Left Tailed Match Pairs	-2.851		-1.773			
3.	Two Tailed Indep. Groups	-1.501		$\pm 2.006$			
4.	Right Tailed Match Pairs	+3.561		+1.692			
5.	Two Tailed Indep. Groups	+0.887		$\pm 1.943$			
6.	Left Tailed Match Pairs	-1.003		-2.759			
7.	Two Tailed Indep. Groups	-4.416		$\pm 1.994$			
8.	Right Tailed Match Pairs	+0.275		+1.839			
9.	Left Tailed Indep. Groups	-1.461		-1.674			
10.	Two Tailed Match Pairs	+2.330		$\pm 2.138$			



**(#11-20) Use each of the following P-values and corresponding significance levels to fill out the table. Assume the data passed all conditions. A low P-value is lower than the significance level. A high P-value is higher than the significance level.**

	P-value Proportion	P-value %	Significance Level Proportion	Significance Level %	Low P-value or High P-value?	If $H_0$ is true, could the sample mean difference or more extreme occur by sampling variability or is it unlikely?	Reject $H_0$ or Fail to reject $H_0$ ?
11.	0.0007		0.1				
12.	0.421		0.01				
13.	$8.71 \times 10^{-5}$		0.05				
14.	0.339		0.01				
15.	0.076		0.05				
16.	0		0.1				
17.	0.528		0.05				
18.	0.0277		0.1				
19.	$3.04 \times 10^{-6}$		0.01				
20.	0.178		0.05				

21. Explain the difference between matched pair data and independent groups.
22. Explain the difference between random samples and random assignment.
23. List the conditions (assumptions) that we need to check for a two-population mean hypothesis test from independent groups.
24. List the conditions (assumptions) that we need to check for a two-population mean hypothesis test from matched pairs.
25. List the conditions (assumptions) that we need to check for a two-population mean hypothesis test that is using experimental design.

**(#26-30) Directions: Use the provided hypothesis test printout and graphs to answer the questions and perform the hypothesis test.**

26. The ACT exam is used by many colleges to test the readiness of high school students for college. Many high school students are now taking ACT prep classes. A local high school offers an ACT prep class, but wants to know if it really helps. Twenty students were randomly selected. They took the ACT exam before and after taking the ACT prep class. For each student the difference between the after and before scores were measured ( $d = \text{after} - \text{before}$ ). Population 1 was the after prep class scores and population 2 was the before prep class scores. The mean of the differences was 1.5 ACT points with a standard deviation of 2.3 ACT points. A histogram of the differences yielded a bell shaped normal distribution. Use a 5% significance level to test the claim that the after prep class scores are higher than the before prep class scores.

N	Sample Mean	Stdev	Significance Level	Critical Value	Test Statistic t	p-Value
20	1.5	2.3	0.05	1.729	2.917	0.0044

- a) Is this two-population mean test matched pair or independent groups?
- b) Write the null and alternative hypothesis. Is the test right tailed, left tailed, or two tailed?
- c) Check all of the conditions for T-test. Does the data pass all of the conditions?
- d) Write a sentence to explain the T-test statistic.



- e) Does the T-test statistic fall in a tail of the distribution starting at the critical value?
- f) Is the sample mean of the differences significantly disagree with zero in the null hypothesis? Explain how you know.
- g) Convert the P-value and significance level into percentages. Is the P-value lower or higher than the significance level?
- h) Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.
- h) Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.
- i) Write the formal conclusion for the hypothesis test.
- j) What does this data indicate about the usefulness of the ACT prep class?

27. A random sample of 20 male German Shepherds found that their average weight was 112 pounds with a standard deviation of 28 pounds. A random sample of 14 male Dobermans found that their average weight is 107 pounds with a standard deviation of 24 pounds. Assume that weights are normally distributed. Use the Statcato printout below and a 5% significance level to test the claim that the population mean average weight of male German Shepherds (population 1) is more than the population mean average weight of male Doberman Pinchers (population 2).

	N	Mean	Stdev
German Shep Sample 1	20	112.0	28.0
Doberman Sample 2	14	107.0	24.0

Significance Level	Critical Value	Test Statistic t	p-Value
0.05	1.697	0.558	0.2906

- a) Is this two-population mean test matched pair or independent groups?
- b) Write the null and alternative hypothesis. Is the test right tailed, left tailed, or two tailed?
- c) Check all of the conditions for T-test. Does the data pass all of the conditions?
- d) Write a sentence to explain the T-test statistic.
- e) Does the T-test statistic fall in a tail of the distribution starting at the critical value?
- f) Do the sample means significantly disagree with each other? Explain how you know.
- g) Convert the P-value and significance level into percentages. Is the P-value lower or higher than the significance level?
- h) Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.
- h) Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.
- i) Write the formal conclusion for the hypothesis test.

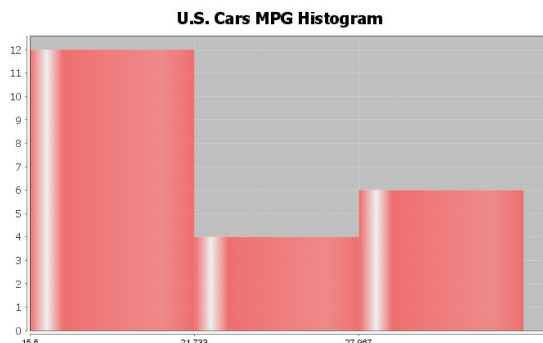


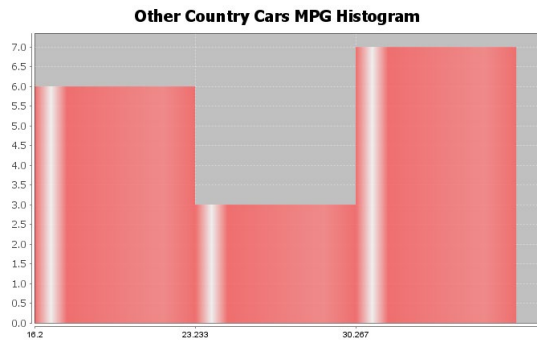
28. Cotinine is an alkaloid found in tobacco and is used as a biomarker for exposure to cigarette smoke. It is especially useful in examining a person's exposure to second hand smoke. A random sample of 32 non-smoking American adults was collected. These adults were not smokers and did not live with any smokers. The average cotinine level for this sample was 7.2 ng/mL with a standard deviation of 5.8 ng/mL. A second random sample of 35 non-smoking American adults was then collected. These adults did not smoke themselves, but did live with one or more smokers. The mean average cotinine level for this sample was 28.5 ng/mL and had a standard deviation of 11.4 ng/mL. Use a 1% significance level to test the claim that people that do not live with smokers have a lower cotinine level than those people that do live with smokers. What does this data indicate about the relationship between cotinine levels and living with a smoker or not.

Significance Level	Critical Value	Test Statistic t	p-Value
0.01	-2.402	-9.758	$1.4751 \cdot 10^{-13}$

- Is this two-population mean test matched pair or independent groups?
- Write the null and alternative hypothesis. Is the test right tailed, left tailed, or two tailed?
- Check all of the conditions for T-test. Does the data pass all of the conditions?
- Write a sentence to explain the T-test statistic.
- Does the T-test statistic fall in a tail of the distribution starting at the critical value?
- Do the sample means significantly disagree with each other? Explain how you know.
- Convert the P-value and significance level into percentages. Is the P-value lower or higher than the significance level?
- Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.
- Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.
- Write the formal conclusion for the hypothesis test.

29. We want to see if cars made in the U.S. have a lower population mean average mpg than those made outside the U.S. We used the random car data at [www.matt-teachout.org](http://www.matt-teachout.org) and a 5% significance level to create the following graphs and statistics with Statcato.





	N	Mean	Stdev
Population 1 USA mpg	22	22.995	6.054
Population 2 Other Country mpg	16	27.188	6.601

\* Population standard deviations are unknown. \*

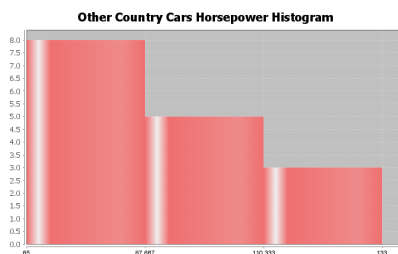
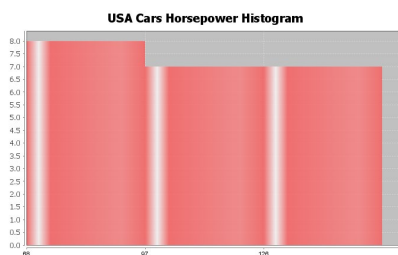
DOF = 30

Significance Level	Critical Value	Test Statistic t	p-Value
0.05	-1.697	-2.001	0.0273

- a) Is this two-population mean test matched pair or independent groups?
- b) Write the null and alternative hypothesis. Is the test right tailed, left tailed, or two tailed?
- c) Check all of the conditions for T-test. Does the data pass all of the conditions?
- d) Write a sentence to explain the T-test statistic.
- e) Does the T-test statistic fall in a tail of the distribution starting at the critical value?
- f) Do the sample means significantly disagree with each other? Explain how you know.
- g) Convert the P-value and significance level into percentages. Is the P-value lower or higher than the significance level?
- h) Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.
- h) Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.
- i) Write the formal conclusion for the hypothesis test.



30. We want to see if cars made in the U.S. have a higher population mean average horsepower than those made outside the U.S. We used the random car data at [www.matt-teachout.org](http://www.matt-teachout.org) and a 10% significance level to create the following graphs and statistics with Statcato. Check the assumptions and perform the hypothesis test.



	N	Mean	Stdev
Population 1 USA car horsepower	22	110.182	26.383
Population 2 Other Country car horsepower	16	90.125	22.408

Significance Level	Critical Value	Test Statistic t	p-Value
0.10	1.306	2.526	0.0081

- Is this two-population mean test matched pair or independent groups?
- Write the null and alternative hypothesis. Is the test right tailed, left tailed, or two tailed?
- Check all of the conditions for T-test. Does the data pass all of the conditions?
- Write a sentence to explain the T-test statistic.
- Does the T-test statistic fall in a tail of the distribution starting at the critical value?
- Do the sample means significantly disagree with each other? Explain how you know.
- Convert the P-value and significance level into percentages. Is the P-value lower or higher than the significance level?
- Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.
- Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.
- Write the formal conclusion for the hypothesis test.



**(#31-32) Directions: Use the given sample statistics and StatKey at [www.lock5stat.com](http://www.lock5stat.com) to answer the questions and perform the hypothesis test.**

31. Use the sample statistics below and a 5% significance level to test the claim that the population mean average pulse rates for women is higher than for men. The data came from a random sample of women's pulse rates in beats per minute (BPM) and a separate random sample of men's pulse rates in beats per minute (BPM). Histograms of both samples have a skewed right shape.

Sample 1 (Women's Pulse Rates)

Sample size = 40

Sample mean = 76.3 BPM

Sample Standard Deviation = 12.5 BPM

Sample 2 (Men's Pulse Rates)

Sample size = 40

Sample mean = 69.4 BPM

Sample Standard Deviation = 11.3 BPM

Two Population Mean Standard Error = 2.664 BPM

Two Population Mean Degrees of Freedom = 77

a) Is this two-population mean test matched pair or independent groups?

b) Write the null and alternative hypothesis.

c) Is this a right tailed, left tailed, or two tailed test? Explain how you know.

d) Check all of the conditions for T-test. Does the data pass all of the conditions?

e) Calculate the T-test statistic with formula  $T = (\text{Sample mean 1} - \text{Sample mean 2}) \div \text{Standard Error}$

Go to the T distribution in the Theoretical Distributions menu in StatKey ([www.lock5stat.com](http://www.lock5stat.com)). Type in the given degrees of freedom. Click on the appropriate tail or tails and put in the significance level proportion in the top proportion box in the tail. The bottom box now shows the critical value where the tail begins.

f) What is the critical value?

g) Does the T-test statistic fall in a tail of the distribution starting at the critical value?

h) Do the sample means significantly disagree with each other? Explain how you know.

Go to the T distribution in the Theoretical Distributions menu in StatKey ([www.lock5stat.com](http://www.lock5stat.com)). Type in the given degrees of freedom. Click on the appropriate tail or tails and put in the T-test statistic in the bottom box in the tail. The top box now shows the P-value.

i) What is the P-value proportion?

j) Convert the P-value into a percentage. Is the P-value lower or higher than the significance level?

k) Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.

l) Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.

m) Write the formal conclusion for the hypothesis test.



32. A random sample of U.S. adults was taken and the difference between their diastolic blood pressure and systolic blood pressures were taken in millimeters of Mercury (mm of Hg). Use the following sample statistics and a 1% significance level to test the claim that the diastolic blood pressure of a person is less than the systolic blood pressure of a person. A histogram of the differences has a skewed left shape.

Sample Blood Pressure Differences (Diastolic - Systolic)

Sample size of Difference (n) = 80

Sample Mean of Differences ( $\bar{d}$ ) = -44.525 mm of Hg

Sample Standard Deviation of Differences ( $s_d$ ) = 10.077 mm of Hg

a) Is this two-population mean test matched pair or independent groups?

b) Write the null and alternative hypothesis.

c) Is this a right tailed, left tailed, or two tailed test? Explain how you know.

d) Check all of the conditions for T-test. Does the data pass all of the conditions?

e) Calculate the Degrees of Freedom  $df = n - 1 = ?$

f) Calculate the Standard Error =  $s_d \div \sqrt{n} = ?$

g) Calculate the T-test statistic with formula  $T = (\text{Sample mean of Differences}) \div \text{Standard Error}$

Go to the T distribution in the Theoretical Distributions menu in StatKey ([www.lock5stat.com](http://www.lock5stat.com)). Type in the given degrees of freedom. Click on the appropriate tail or tails and put in the significance level proportion in the top proportion box in the tail. The bottom box now shows the critical value where the tail begins.

h) What is the critical value?

i) Does the T-test statistic fall in a tail of the distribution starting at the critical value?

j) Do the sample means significantly disagree with each other? Explain how you know.

Go to the T distribution in the Theoretical Distributions menu in StatKey ([www.lock5stat.com](http://www.lock5stat.com)). Type in the given degrees of freedom. Click on the appropriate tail or tails and put in the T-test statistic in the bottom box in the tail. The top box now shows the P-value.

k) What is the P-value proportion?

l) Convert the P-value into a percentage. Is the P-value lower or higher than the significance level?

m) Could the T-test statistic or more extreme have occurred by random chance (sampling variability) or is that unlikely? Explain how you know.

n) Should we reject the null hypothesis (significant evidence) or fail to reject the null hypothesis (not significant evidence)? Explain your answer.

o) Write the formal conclusion for the hypothesis test.

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